CONTRIBUTIONS TO RECENT FRONTIERS IN TRIBOLOGY

By

MOHAMMAD ISA

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MARCH 1974

CONTRIBUTIONS TO RECENT FRONTIERS IN TRIBOLOGY

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

By

MOHAMMAD ISA

to the

DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MARCH 1974

CERTIFICATE

This is to certify that the matter embodied in the thesis entitled "Contributions to Recent Frontiers in Tribology" by Mr Mohammad Isa for the award of the Degree of Doctor of Philosophy of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under my supervision and guidance The thesis has, in my opinion, reached the standard fulfilling the requirements of the Ph D degree The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma

Dated March 14, 1974

[J B Shukla]
Thesis Supervisor

ACKNOWLEDGEMENTS

I shall ever remain deeply indebted and extremely grateful to my supervisor, Dr J B Shukla who has guided me throughout this work and without whose invaluable inspiration and encouragement the investigations contained in this thesis would not have been possible

I am grateful to Aligarh Muslim University for deputing me, under Quality Improvement Programme, to work for Ph D Degree at the Indian Institute of Technology, Kanpur I wish to thank, the Ministry of Education, Government of India, for providing me financial assistance in carrying out this research work I am also grateful to Professor N C Saha and Professor N A Khan for their constant inspiration in this work

My thanks are also due to Mr Peeyush Chandra and other members of Tribology and Biomechanics Group for their generous and helpful discussions

I wish to thank Mr G L Misra and Mr S K Tiwari for their unfailing patience in type-cutting the stencils and Mr A N Upadhyaya for neatly cyclostyling them The tremendous amount of pains Mr B N Srivastava has taken in drawing figures is also gratefully acknowledged

Finally, I must express my indebtness to my wife 'Akbari' for her great help and forebearance during the preparation of this work

Mohammad Isa

Dated March 14, 1974 *(Mohammad Isa)*

To Mozaffar Bhai Marhoom

CONTENTS

SYNOPSIS

OF

THE THESIS ENTITLED

"CONTRIBUTIONS TO RECENT FRONTIERS IN TRIBOLOGY"

Tribology is the name of an interdisciplinary science which signifies and covers all aspects of problems related to lubrication, friction and wear in its totality, even if parts of the problem belong to various disciplines of Engineering and Sciences In this thesis some original contributions to Tribology are embodied The study has been mainly confined to the following aspects of lubrication in various tribological systems

(1) Effects of additives in fluid film lubrication

(2) Non-Newtonian power law fluid lubrication

(3) Magnetohydrodynamic lubrication

The contributions have been described in ten chapters as follows

In Chapter I, a survey of the recent contributions on the above three aspects has been outlined and the prospective of the present work is indicated

In Chapter II, the generalized Reynolds equation for lubricants, containing additives which can be represented as micropolar fluids, is derived and its application to optimum one dimensional slider bearing has been studied by using calculus of variation It has been shown that the load capacity and friction force increase and the coefficient of friction decreases as the parameter characterising the micro-structure of the base oil due to presence of additive increases

In Chapter III the above theory of micropolar fluid lubrication has been applied to an externally pressurised bearing by using variational approach The optimum film thickness profile is obtained for maximum load capacity of the bearing The effects of additives on the various characteristics of this optimum bearing have been investigated

In Chapter IV the effects of viscosity variation, due to change in concentration of the additive in the base oil on the characteristics of slider and externally pressurised bearings have been studied It has been shown that both the load capacity and friction force increase as the concentration of the additive in the base oil increases

Chapter V deals with the characteristics of non-Newtonian power law lubricants in an externally pressurised conical step bearing and in a hydrostatic step seal It has been pointed out that the step in the film thickness contributes to the increase in the load capacity of the above systems The effect of flow behaviour index of the power law fluid on the characteristics of these tribological systems is also studied

In Chapter VI the thermal effects in externally pressurised bearings using a non-Newtonian power law lubricant have been studied by considering the consistency of the lubricants as a function of temperature and pressure It has been shown that the load capacity of the bearings decreases as the parameters characterising the change in consistency increase Further, in the case of externally pressurised circular bearing the load capacity increases as the flow behaviour index decreases, but the load capacity in the case of rectilinear thrust plates does not depend upon this index The thermal effects on the flow flux of the bearing are also investigated

In Chapter VII, the characteristics of non-Newtonian power law lubricants in the case of externally pressurised porous thrust bearing have been treated It is found that the load capacity increases as the flow behaviour index or the parameter characterising the porosity of the bearing increases

Chapter VIII is devoted to the study of squeeze film between rectangular and circular plates using power law lubricants where the effects of inertia, pressure and step in film thickness have been considered It has been shown that the effect of inertia and pressure is to increase the load capacity of the bearing, while the effect of step in the film thickness is to decrease the load capacity Similar effects have been found on the squeezing time in both the above cases

In the last two chapters certain tribological aspects of magneto-hydrodynamic lubrication are studied in externally pressurised bearings

In Chapter IX, the characteristics of an externally pressurised bearing using conducting incompressible lubricant in the presence of non-uniform axial magnetic field have been investigated by using variational approach It has been shown that for maximum load capacity of the bearing both the film thickness and the magnetic field profiles should be step functions This maximum load increases as the strength of the applied magnetic field increases

In Chapter X, the magnetohydrodynamic externally pressurised porous thrust bearings have been studied in the presence of uniformly applied transverse magnetic field It has been shown that the load capacity increases as the supply pressure, viscous permeability or the strength of the applied magnetic field increases The effects of squeezing have also been studied and it has been pointed out that the load capacity increases as the squeeze velocity increases and this increase is enhanced as the strength of the applied magnetic field increases

LIST OF SYMBOLS

b	=	$\frac{r_e}{r_i}$ in Chapter IX otherwise it refers to the width of the slider bearing
B		strength of the magnetic field
C		volume concentration of the suspension
C_o		volume concentration of the base oil
c_f		dimensionless coefficient of friction
c_v		specific heat at constant volume
D		diffusion coefficient
e		exponential function
E		Weierstrass excess function
F_m		friction force at the slider for maximum load capacity
h		film thickness
h_o		minimum film thickness
h_1, h_2		film thickness in regions I and II respectively
h_m		maximum film thickness corresponding to optimum profile
H	=	h_1/h_2 in Chapter V otherwise thickness of the porous surface
I_o, K_o		Modified Bessel functions of order zero of first and second kind respectively
I_1		modified Bessel function of order one of first kind
k, k_1		rate of mass transfer at the surfaces (in Chapter IV)
k		thermal conductivity (in Chapter VI)
k		step location
k'		rate of chemical reaction
L		length of the slider bearing
m		consistency index of the power law fluid
m_o		constant consistency index

M	torque in hydrostatic bearing (in Chapter IV)
M	Hartmann number = $Bh(\frac{\sigma}{\mu})^{1/2}$ (in Chapters IX and X)
n	flow behaviour index of the power law fluid
p	pressure in the thin film of the lubricant
p_a	exit or ambient pressure
p_e	externally applied pressure
p_i	inlet pressure
p_m	maximum pressure
p_s	pressure at the step
p_1 p_2	pressures in region I and II respectively
p'	pressure in the porous matrix
Q	Flow rate or flux
Q_o	volume flux with uniform film thickness, or constant consistency index
Q'	volume flux in the porous capillary
Q_1, Q_2	flux in region I and II respectively
Q_x Q_z	volume fluxe in x and z directions respectively
r	radial variable
r_e	outer radius
r_i	inlet radius
r_o	step position (in Chapter III)
R =	$\frac{r_e}{r_i}$ in Chapter III otherwise it refers to outer radius
R_o	inlet radius (in Chapter IV)
R_a	average radius of the porous capillary
t	squeezing time
T	frictional force at the lower plate in slider bearing (in Chapter IV)

T		temperature (in Chapter VI)
u		velocity in x or r direction
U		velocity of the slider
v		velocity in y direction
v'		velocity in the porous capillary
V		squeeze velocity
$\bar{W}$, $\bar{W}_1$ $\bar{W}_1'$ $\bar{W}_c$ $\bar{W}_s$ $\bar{W}_m$		dimensionless load capacities as defined in the analysis
x	=	$\frac{r}{r_i}$ in Chapters III and IX otherwise it refers to a variable in Cartesian coordinate system
x_o	=	$\frac{r_o}{r_i}$ (in Chapter III)
y		co-ordinate normal to the plane of the film
Y	=	$\frac{h}{h_o}$ dimensionless film thickness
Y_o	=	$\frac{h_m}{h_o}$
z		rectangular coordinate
α_v β_v, γ		material constants (in Chapters II and III)
α		semivertical angle of the cone (in Chapter V)
γ	=	$\rho\ c_v$ (in Chapter VI)
δ		operator for variation
μ		viscosity
μ_o		viscosity of the base oil
μ_1		viscosity of the liquid dispersed in the lubricant forming emulsion (in Chapter IV), material constant (in Chapters II and III)
$\bar{\mu}_1$	=	$\frac{\mu_1}{\mu}$ an index to the microstructure character of the micropolar model
ρ		fluid density
σ		conductivity
τ_{xy}, τ'_{ry}		shear stress in the lubricant and porous capillary respectively
Ω		angular velocity of the rotor (in Chapter IV)
ω		microrotation vector

CHAPTER - I

INTRODUCTION

1 1 WHAT IS TRIBOLOGY ?

In general, most of the lubricated systems can be considered to consist of two relatively moving surfaces (plane or curved, loaded or unloaded) with a thin film of external material (lubricant) between them The presence of such a thin film between these surfaces not only helps them to support considerable load but also minimises friction

Since the days of Osborne Reynolds [1886] , various mathematical models have been proposed to study the characteristics of such systems under various simplifications, Cope [1949] , Dowson [1962] and Cameron [1966] However, to cope with the complexities of modern man-and-machine systems which arise in space travel, nuclear reactors etc , the need for new frontiers of research involving lubrication, friction and wear has arisen To study such problems in a more realistic and comprehensive manner, the knowledge of the following is desired

(1) Nature of the surfaces, such as roughness, elasticity, thermal conductivity, hardness, porosity, and its affinity such as adsorption, absorption, chemical reaction etc with the lubricant

(2) Nature of the lubricant solid, liquid or gas, Newtonian or non-Newtonian behaviour, variation of viscosity or consistency with temperature, pressure or concentration and its affinity with the surface

(3) Effects of surrounding environment, external and internal forces etc

It is obvious from the above that to have the total picture of the problem, the knowledge of various disciplines of Science and Engineering is required In particular, if in the lubricated system, adsorption of the lubricant is taking place on the metallic surface due to chemical reaction, then the knowledge of Chemical Engineering, Chemistry, Metallurgy and Physics etc , is involved Further, if the surfaces are rough, then to study their topography due to randomness of roughness, the knowledge of Statistics, Production Engineering and Metal Physics etc , is required If the system forms a part of human body such as a human joint, the knowledge of Biology, Medicine and Engineering is needed. To make a comprehensive mathematical model of the system, the help of a mathematician is also desirable Thus to have a realistic investigation of such problems, a combined effort by scientists and engineers specialized in various aspects of the problem is needed Such an attempt did not come forth till the middle of this century, and an effort was needed to generate such interactions in the field of lubrication, friction and wear Keeping this in view, the Ministry of Technology, United Kingdom, appointed a committee known as Jost Committee, in the late sixtees, to investigate the short comings in the area of research in the above fields and its eventual application in the industry It is this committee which, in fact, coined the new name, TRIBOLOGY, which signifies and covers all aspects of the problem related to lubrication, friction and wear in its totality, even if parts of the

problem belong to many usual disciplines, such as Mechanical Engineering, Industrial Engineering, Chemistry, Chemical Engineering, Physics, Metallurgy, Textile, Leather, Medical sciences etc Thus, tribology is meant to be truly interdisciplin ary in nature and is mainly problem oriented in the real sense of the term, Shukla [1972]

In this thesis an attempt is made to study the characteristics of certain tribological systems by using thin film lubrication theory The following three aspects have been investigated

(1) Effects of additives in fluid film lubrication

(2) Non-Newtonian Power law fluid lubrication

(3) Magnetohydrodynamic lubrication

1 2 EFFECTS OF ADDITIVES IN FLUID FILM LUBRICATION

It is needless to emphasize the importance of additives in lubricants because of the diverse and highly stringent environment (such as in, missiles, air crafts and other space vehicles) to which they are subjected Consequently, a specialized knowledge is required for careful selection of additives for each individual application, Brazier [1965] In particular, to improve the characteristics of the base oil, the additives can be used as rust inhibitors (Amine phosphates), corrosion inhibitors(sulphurised olefines), fire resistant (halogenated hydrocarbons), detergents (Calcium/Barium Sulphonate), viscosity improvers (polymethacrylate, powders of graphite and molybdenum disulfide), E P additive (sulphurised fats), Molyneux [1967] Even additives in gases have also been used and the process is known as aerosol lubrication, Carr et al [1965]

Several experimental investigations have been carried out by using different additives in the base oil and the reduction in wear rate and coefficient of friction have been pointed out, McCabe [1965], Stock [1966], Molyneux [1967], Rowe [1970], Bartz [1971] In particular, Talivaldis Spalvins [1971] has carried out an experiment and has shown that thin sputtered molybdenum disulfide film on highly polished metal surface gives low average coefficient of friction and long wear life at a particular load Gansheimer [1972] has recently conducted an experiment on the lubricating properties of the mixture of mineral oil with certain inorganic compounds and has pointed out that the mineral oil suspensions of iron and zinc pyrophosphates have high load carrying ability and good antiwear properties

However, very little attention has been given to develope a mathematical model predicting the effects of additives in hydrodynamic lubrication where the viscosity of the suspension has been taken into account Herzig [1959] has given a mathematical relation between the coefficient of friction and film thickness in the case of foil bearing when molybdenum disulfide has been added to the lubricant This relation has been obtained by comparing the theoretical results in the case of foil bearing with the experimental data obtained by adding additive to the oil It has been shown that the coefficient of friction depends upon a factor which is a function of concentration and size of the particles

Since, viscosity is the most important factor governing hydrodynamic lubrication, the effects of additives in lubrication may, in

general, be studied theoretically by a mathematical model involving viscosity of the base oil, concentration of the additive, nature of the additive (solid or liquid), geometry of the particles of the additive in the case of solids, their interaction with the base oil etc Einstein [1906] has been the first who has given a mathematical formula for the apparent viscosity of the fluid containing additive in terms of its concentration for dilute suspension He has pointed out that as the concentration of the additive increases, the viscosity of the suspension increases Since then a number of generalizations of the above relation has been proposed in the case of suspensions and emulsions for dilute or low concentrations, Rutger [1962] , Sherman [1962] Also, Seshadri and Sutera [1968] have pointed out that the concentration of the suspended particles change when suspension flows and the concentration is governed by the equation of mass transfer and depends upon convection, chemical reaction, size of the particles, geometry of the system etc Thus, in most general form, the apparent viscosity of the suspension can be represented as a series involving powers of concentration of the additive which may change with chemical reaction, size of the particles and the geometry of the system etc

Further, in the case of physical interaction of the additives with the base oil, such as micro-rotation of the solid particles, the flow and lubrication behaviour of the suspension may be studied by considering them as micropolar fluids, Eringen [1965] , Hudimoto and Tokuoka [1969] , Allen and Kline [1971] , Datta [1972] , Agarwal et al [1972] , Khader and Vachon [1973] In addition, in the case of

polymer additives, if the concentration increases from a certain value, the suspension no longer behaves as a Newtonian fluid but follows the non-Newtonian character [see next section]

In view of this, we have considered, in Chapters II and III of this thesis, the effects of solid particles in the lubricant by characterizing it as a micropolar fluid In chapter II, the generalised Reynolds equation for micropolar lubricants has been derived and the optimum one dimensional slider bearing is studied It is found that both the maximum load capacity and the corresponding frictional force increase as the parameter, characterizing the micro-structure of the base oil due to the presence of additive, increases, but the coefficient of friction decreases Similarly, in Chapter III, the case of an externally pressurised bearing with micropolar lubricant has been studied by using the techniques of calculus of variation and it has been pointed that the maximum load capacity increases as the step height ratio or the parameter characterizing the microstructure of the suspension increases The effects of these parameters on the flow flux are also studied

In Chapter IV, the effects of additives have been considered by assuming the apparent viscosity of the suspension as a linear function of concentration of additives by taking mass transfer parameter into account The cases of slider and externally pressurised bearings have been studied and it has been shown that the load capacity and frictional force increase as the concentration of the additive in the base oil or the mass transfer parameter increases

1 3 NON-NEWTONIAN POWER LAW FLUID LUBRICATION

In the case of motion of a fluid, when the shear stress is proportional to the rate of shear, the fluid is said to follow Newtonian hypothesis and is termed as Newtonian fluid Those fluids which do not follow this Newtonian behaviour are called non-Newtonian Strictly speaking most of the fluids express non-Newtonian behaviour when subjected to precise measurement, Pinkus [1961] Mathematically, the non-Newtonian fluids could be characterized by the following constitutive relation:

Shear stress = consistency x rate of shear

where consistency can, in general, be a function of both shear stress and shear rate Depending upon this relation various types of non-Newtonian models such as Bingham plastic, viscoelastic, power law etc , have been proposed, Bird et al [1960] , Longwell [1966] Non-Newtonian behaviour is almost invariably observed in solutions or melts of high-molecular-weight polymeric materials and suspensions of solids in liquid Some of the examples of non-Newtonian fluids are: suspension of chalk, paints, printing ink, greases, methacrylate in water, detergent slurries etc , Bird et al [1960] , Longwell [1966]

In the last two decades or so, various investigators have studied the lubrication characteristics of these fluids in various bearings , Pinkus [1960] In particular, the characteristics of Bingham plastic lubricants (greases) have been studied in the case of slider bearing, Milne [1953, 1957] , squeeze films and hydrostatic thrust bearing,

Osterle et al [1955] and Batra [1973] , roller bearings, Sasaki et al [1962] Kauzlarich and Greenwood [1972] have considered a generalized Bingham model known as Herschel Bulkley Model and have studied the concept of elastohydrodynamic lubrication for greases The MHD-thrust bearing using Bingham model has also been studied by Shukla and Prasad [1967]

The viscoelastic fluids as lubricants have been studied by Milne [1957a] , Selby [1958], Horowitz and Steidler [1960-61] , Wright and Crouse [1965], Ghosh and Mishra [1968], Tanner [1970], Hung and Muster [1970],in various bearings Chow and Saibel [1971] have studied analytically the film thickness and pressure distribution for a heavily loaded line contact of rollers using viscoelastic lubricants and have pointed out a reduction in the film thickness from its Newtonian value

The behaviour of Power law lubricants has been studied by several investigators Ng and Saibel [1962] have investigated the use of a modified model of the pseudoplastic power law lubricant in the case of an inclined slider bearing and have shown that the load capacity is less than that of Newtonian lubricant Hsu and Saibel [1965] have studied the case of a slider bearing by using a cubic model and have pointed out that the load capacity and friction force are reduced but the flow rate is comparatively increased in comparison to Newtonian case Tanner [1963, 1964, 1965] has studied the non-Newtonian lubrication theory and has applied this to a short journal bearing by using power law lubricants The use of Power law fluids in squeeze films and externally pressurised conical bearings have also been studied, Shukla [1963a, 1964-b, 1964-c]

In the case of conical step bearing, it has been pointed out that the load capacity increases as the flow behaviour index decreases Shukla and Prakash [1969] have also studied the use of power law fluid in the case of rheostatic thrust bearing and have shown that, for the maximum load capacity of the bearing, the film thickness should be a step function

The characteristics of non-Newtonian power law lubricants in journal bearings have been investigated theoretically by Tanner [1964, 1965] and Hsu [1966] and experimentally by Dubois et al [1960] Hsu [1966] has obtained solution applicable to both pseudoplastic and dilatant non-Newtonian fluids, which are valid over a wide range of shear stress

In view of the above, it may be noted that the effects of step in film thickness, consistency variation with temperature and pressure, porosity, elasticity, roughness and inertia, in various bearings have not been considered for non-Newtonian lubricants, Dowson et al [1966], Castelli et al [1966], Christensen [1969-71] etc In this thesis, some of the above effects have been studied in externally pressurised and squeeze film bearings for power law lubricants only

In Chapter V, the effects of step in film thickness on the various characteristics of externally pressurised conical bearing and hydrostatic step seal have been investigated It is shown that load capacity of these bearings increase with the increase in step height It is also seen that in the case of a hydrostatic step seal, the load capacity

increases as the flow behaviour index of the fluid increases

In Chapter VI, the effect of temperature on the consistency index of the lubricant is considered in the case of hydrostatic thrust bearings (rectilinear and circular) By considering an exponential variation of consistency with temperature and pressure, it is shown that the load capacity decreases as the flow behaviour index or the parameter characterizing the consistency variation increases

In Chapter VII, the characteristics of externally pressurised porous thrust bearing with power law lubricants have been studied and it has been shown that the load capacity increases as the flow behaviour index or the parameter characterizing the porosity increases

In Chapter VIII, the characteristics of squeeze film bearings with power law lubricants are studied by taking inertia and the linear variation of consistency with pressure into account It has been shown that the effect of inertia and pressure is to increase the load capacity of the bearing when the film thickness is constant Further, in the absence of inertia and pressure effect, it has been shown that the load capacity and time of approach decrease as the step height increases

1 4 MAGNETOHYDRODYNAMIC LUBRICATION

Magnetohydrodynamic or hydromagnetic lubrication is the study of lubrication process of electrically conducting lubricants in the presence of electric or/and magnetic fields Because of high temperature applications such as in nuclear reactors and space vehicles, the use of liquid metals as lubricants has been suggested Since the liquid

metals are less viscous than oil and are good electrical conductors, its lubricating property can be improved by applying external electro-magnetic fields in the system In fact, when a conducting fluid flows in a bearing and a magnetic field is applied in a transverse direction to motion, a body force is generated which modifies the lubrication characteristics of the system In rec nt years, several scientists and engineers have contributed in the field of hydromagnetic lubrication by keeping their objective around the possibility of using different magnetic and electric field geometries to get an increase in the pressurization of liquid metal bearings

In the early sixtees, Fucks and Uhlenbush [1962] and Snyder [1962] have studied the characteristics of MHD slider bearings and have indicated that the load capacity can be increased by using conducting lubricant in the presence of externally applied magnetic field Since then several investigators have investigated the characteristics of MHD slider bearings (finite or infinite) in the presence of transverse and tagential magnetic fields, Elco and Hughes [1962], Hughes [1963], Shukla [1964a], Chawla [1966], Prak sh [1967], Ramanaiah [1966], Kapur [1969] and Agarwal [1970] A generalized Reynolds equation for hydromagnetic lubrication has also been derived by Shukla [1963] In addition, the calculus of variation approach has been used to study the optimum MHD slider bearing by Osterle and Young [1962], Kuzma [1965], Shukla [1970], and it has been pointed out that for maximum load capacity of the bearing the film thickness should be a step function The characteristics of MHD slider bearing with non uniform magnetic field have been discussed by Rodkiewicz and Anwar [1972, 1972a]

Several theoretical and experimental investigations have been conducted to study the characteristics of MHD journal bearings in the presence of radial as well as axial magnetic fields, Hughes and Elco [1962a], Kuzma [1963, 1964b], Shvarts [1966], Kamiyama [1969a], Kamiyama and Sato [1971] Dudzinsky et al [1968] have presented an analysis and experimental results for a MHD partial journal bearing and have shown that there exists an optimum Hartmann number at which the load capacity attains a maximum for a given applied current

The problem of MHD squeeze film has been studied by Kuzma et al [1964], Kuzma [1964a], Shukla [1965], Shukla and Prasad [1965, 1967], Shukla and Prakash [1966], and the increase in load capacity and time of approach due to increase in magnetic field have been pointed out

The characteristics of externally pressurised MHD bearings have been studied by Hughes and Elco [1962], Shukla and Prasad [1966], Kriegar et al [1967], Maki et al [1966,1967], Kapur [1968], Chow [1969] Shukla [1965a] has studied the MHD externally pressurised bearing with variable film thickness and has shown that the load capacity is greater than that of a bearing having a constant film thickness The effects of inertia in MHD hydrostatic thrust bearing have also been investigated, Ramanaiah [1967], Shukla and Kapur [1967] Kamiyama [1969, 1970, 1971], Kamiyama and Sato [1972a, 1972b], in a series of papers, have studied the inertia effects and influence of wall conductance on the characteristics of MHD hydrostatic thrust bearings

In view of the above, in Chapter IX, the characteristics of an externally pressurised bearing using conducting incompressible lubricant in the presence of a non-uniform axial magnetic field have been investigated by using variational approach It has been shown that, for maximum load capacity of the bearing, both the film thickness and the magnetic field profiles should be step functions It has also been indicated that this maximum load increases as the strength of the magnetic field, applied in the region of minimum film thickness, increases

In Chapter X, the magnetohydrodynamic externally pressurised porous thrust bearings have been studied in the presence of uniformly applied transverse magnetic field It has been shown that the load capacity increases as the supply pressure, viscous permeability or the strength of the applied magnetic field increases The effects of squeezing have also been studied and it has been pointed out that the load capacity increases as the squeeze velocity increases and this increase is enhanced as the strength of the applied magnetic field increases

CHAPTER - II

GENERALISED REYNOLDS EQUATION FOR MICROPOLAR LUBRICANTS AND ITS APPLICATION TO OPTIMUM ONE DIMENSIONAL SLIDER BEARING EFFECTS OF ADDITIVES IN LUBRICATION

The use of additives, in the form of solid particles, in the base oil to improve its lubrication characteristics in bearings, is well known However, very little attention has been given to predict these characteristics by considering the mathematical model which takes into account the presence of additives in the base oil Einstein [1906] has been the first who pointed out that viscosity, the most important characteristic of the fluid, increases as the concentration of the additive increases For low concentration of the additive the suspension behaves as a Newtonian fluid, but for very high concentration of the additive the suspension no longer follows Newtonian hypothesis and it can only be characterised by a non-Newtonian model In addition, the solid particles present in the fluid, apart from moving with the usual velocity of the lubricant might have micro-rotation in the fluid elements Such suspensions are represented by micropolar fluids Eringen [1965] , Allen and Kline [1971] , Khader and Vachon [1973] In this chapter, this particular model of suspension is being considered to study the effects of additives in lubrication The generalised Reynolds equation is derived and the characteristics of optimum one dimensional slider bearing are studied by using techniques of calculus of variation , Rayleigh [1918] , Maday [1968] and Shukla [1970]

2 1 GENERALISED REYNOLDS EQUATION

The basic equations governing the flow of a fluid containing additives represented as micropolar fluid are given by Eringen [1966]

$$\rho[\frac{\partial V}{\partial t} - V \times (\nabla \times V) + \frac{1}{2}\nabla(V^2)]$$

$$= (\lambda+2\mu+\mu_1)\nabla\nabla V \quad (\mu+\mu_1)\ \nabla \times \nabla \times V + \mu_1\ \nabla \times \bar{\omega} \quad \nabla p \tag{2 1}$$

$$\rho j\ \omega = (\alpha_v + \beta_v + \gamma)\ \nabla\nabla\ \omega \quad \gamma\nabla \times \nabla \times \bar{\omega} + \mu_1\ \nabla \times V \quad 2\mu_1\ \omega \tag{2 2}$$

$$\nabla\ V = 0\,, \tag{2 3}$$

where body forces and body moments are neglected The various symbols have been defined in the nomenclature and are not described here

Let us consider now the flow of the lubricant in a thin film enclosed by two relatively moving surfaces The film thickness is negligibly small in comparison to the other dimensions of the surfaces, so that the usual hydrodynamic lubrication assumptions are applicable , Pinkus and Sternlicht [1960] The physical situation is described in figure no (2 1)

To derive the Reynolds equation for this lubricant,let us consider, as usual, the following components of velocity and micro-rotation vectors

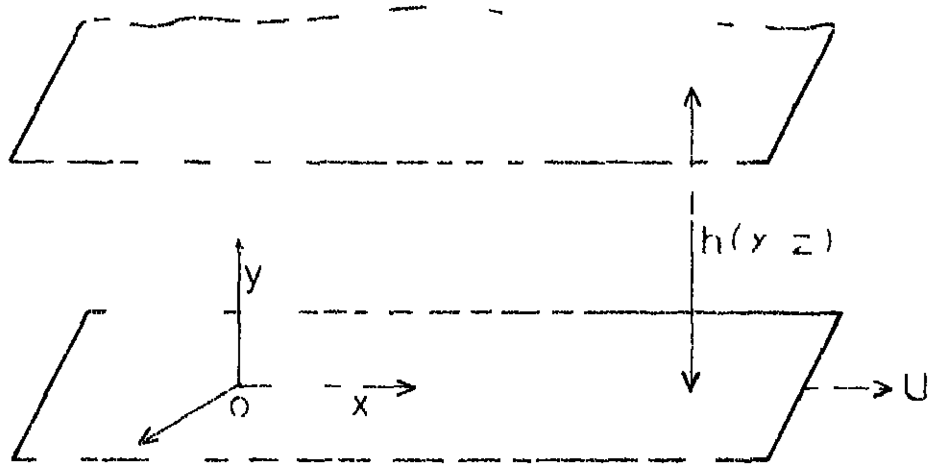


FIG 2 1 FLOW OF A MICROPOLAR FLUID IN A THIN FILM

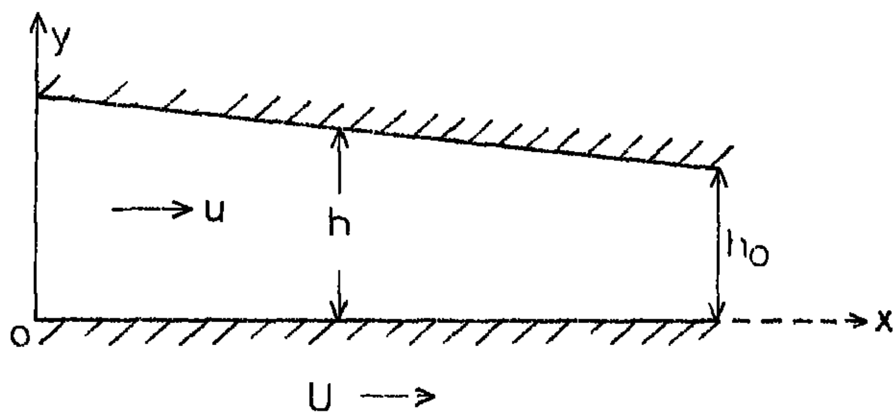


FIG 2 2 ONE DIMENSIONAL SLIDER BEARING WITH MICROPOLAR LUBRICANTS

$$V = (u_1 \quad u_2 \quad u_3)$$

$$\bar{\omega} = (\omega_1 \quad 0 \quad \omega_3) \tag{2 4}$$

where the components of velocity and micro-rotation vectors are assumed small in the direction normal to the bearing surfaces [$u_2 << u_1, u_3$] Using the usual assumptions of lubrication theory [neglecting spin inertia etc] , from equations (2 1) to (2 4) we get the following equations governing the flow of the lubricant in the film

$$(\mu+\mu_1) \frac{\partial^2 u_1}{\partial y^2} + \mu_1 \frac{\partial \omega_3}{\partial y} - \frac{\partial p}{\partial x} = 0 , \tag{2 5}$$

$$(\mu+\mu_1) \frac{\partial^2 u_3}{\partial y^2} - \mu_1 \frac{\partial \omega_1}{\partial y} \quad \frac{\partial p}{\partial z} = 0 , \tag{2 6}$$

$$\gamma \frac{\partial^2 \omega_3}{\partial y^2} - \mu_1 \frac{\partial u_1}{\partial y} \quad 2\mu_1 \omega_3 = 0 \tag{2 7}$$

$$\gamma \frac{\partial^2 \omega_1}{\partial y^2} + \mu_1 \frac{\partial u_3}{\partial y} \quad 2\mu_1 \omega_1 = 0 \tag{2 8}$$

The boundary conditions for the above system of equations are chosen as follows:

$$u_1 = U \quad \text{at} \quad y = 0, \quad u_1 = 0 \quad \text{at} \quad y = h \tag{2 9}$$

$$u_3 = 0 \quad \text{at} \quad y = 0, \quad y = h \tag{2 10}$$

$$\omega_1 = 0 \quad \text{at} \quad y = 0, \quad y = h \tag{2 11}$$

$$\omega_3 = 0 \quad \text{at} \quad y = 0, \quad y = h \tag{2 12}$$

Solving equations (2 5) and (2 7) and using the boundary conditions (2 9) and (2 12) we obtain the following expressions for u_1 and ω_3,

$$u_1 = U + A_1[\beta \sinh \lambda y + 2y] + B_1[\beta (\cosh \lambda y - 1)] + \frac{y^2}{2\mu+\mu_1} \frac{\partial p}{\partial x} , \quad (2\ 13)$$

$$\omega_3 = A_1(\cosh \lambda y - 1) + B_1 \sinh \lambda y - \frac{y}{2\mu+\mu_1} \frac{\partial p}{\partial x} , \quad (2\ 14)$$

where $A_1 = [\frac{h}{2(2\mu+\mu_1)} \frac{\partial p}{\partial x} + \frac{U}{2} \frac{\sinh \lambda h}{\{\beta(\cosh \lambda h - 1) + h \sinh \lambda h\}}]$,

$$B_1 = \frac{1}{\sinh \lambda h} [\frac{h}{2\mu+\mu_1} \frac{\partial p}{\partial x} - A_1(\cosh \lambda h - 1)] ,$$

$$\lambda^2 = \frac{(2\mu+\mu_1)\mu_1}{(\mu+\mu_1)\gamma} \qquad \beta = \frac{\lambda\gamma}{\mu_1} - \frac{2}{\lambda}$$

Similarly from equations (2 6) and (2 8) with boundary conditions (2 10) and (2 11) we get the expressions for u_3 and ω_1 as follows

$$u_3 = A_2[\beta \sinh \lambda y + 2y] + B_2 \ \beta(\cosh \lambda y - 1) + \frac{y^2}{2\mu+\mu_1} \frac{\partial p}{\partial z} \quad (2\ 15)$$

$$\omega_1 = A_2 (\cosh \lambda y - 1) + B_2 \sinh \lambda y - \frac{y}{2\mu+\mu_1} \frac{\partial p}{\partial z} \quad (2\ 16)$$

where

$$A_2 = \frac{h}{2(2\mu+\mu_1)} \frac{\partial p}{\partial z} ,$$

$$B_2 = \frac{1}{\sinh\lambda h} [\frac{h}{2\mu+\mu_1} \frac{\partial p}{\partial z} - A_2(\cosh \lambda h - 1)]$$

The flow flux per unit width in the x and z directions are defined by the following equations

$$Q_x = \int_0^h u_1 dy \tag{2 17}$$

$$Q_z = \int_0^h u_3 dy \tag{2 18}$$

which on using equations (2 13) and (2 15) give

$$Q_x = \frac{Uh}{2} - \frac{h_o^3}{6(2\mu+\mu_1)} \frac{\partial p}{\partial x} G(\frac{h}{h_o}) , \tag{2 19}$$

$$Q_z = - \frac{h_o^3}{6(2\mu+\mu_1)} \frac{\partial p}{\partial z} G(\frac{h}{h_o}), \tag{2 20}$$

where $G(\bar{h}) = G(\frac{h}{h_o}) = \bar{h}^3 + \frac{3\mu_1 h}{2(\mu+\mu_1)} \frac{1 - \alpha_1\bar{h} \coth \alpha_1 \bar{h}}{\alpha_1^2}$,

$$\alpha_1^2 = \frac{\lambda^2 h_o^2}{4} = \frac{M_1 \mu_1(2\mu+\mu_1)}{\mu(\mu+\mu_1)} \text{ and } M_1 = \frac{h_o^2 \mu}{4\gamma}$$

On integrating the equation of continuity (2 3) we have

$$\frac{\partial}{\partial x}\int_0^h u_1\,dy + \frac{\partial}{\partial z}\int_0^h u_3\,dy = 0 \tag{2 21}$$

and using equations (2 17) to (2 20) we get the generalised form of Reynolds equation for micropolar lubricant as follows

$$\frac{\partial}{\partial x}\left[\frac{h_0^3}{6(2\mu+\mu_1)}\,G(h)\,\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{h_0^3}{6(2\mu+\mu_1)}\,G(h)\,\frac{\partial p}{\partial z}\right] = \frac{U}{2}\,\frac{\partial h}{\partial x} \tag{2 22}$$

It is this equation which determines the pressure in the film for a given function h

Since $\dfrac{1 - \alpha_1 \bar{h}\coth \alpha_1 h}{\alpha_1^2} \rightarrow \dfrac{\bar{h}^2}{3}$

as $\mu_1 \rightarrow 0$, $\alpha_1 \rightarrow 0$, the function $G(\bar{h})$ tends to $\bar{h}^3$ Then for $\mu_1 = 0$, equation (2 22) reduces to the following form

$$\frac{\partial}{\partial x}\left\{\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right\} = \frac{U}{2}\frac{\partial h}{\partial x} \tag{2 23}$$

This is the well known Reynolds equation for Newtonian fluids

2 2 OPTIMUM ONE DIMENSIONAL SLIDER BEARING

In this section, we study the characteristics of micropolar lubricants in the case of one dimensional slider bearing whose configuration is shown in figure no (2 2), Allen and Kline [1971] ,

Balram and Shastry [1972] , Datta [1972]

In this case, the generalised Reynolds equation (2 22) can be written as

$$\frac{d}{dx}\left\{\frac{h_o^3}{6(2\mu+\mu_1)} G(h) \frac{dp}{dx}\right\} = \frac{U}{2}\frac{dh}{dx} \qquad (2\ 24)$$

The flow flux $Q(= Q_x)$ is given from equation (2 19) as

$$Q - \frac{Uh}{2} \quad \frac{h_o^3}{6(2\mu+\mu_1)} \frac{dp}{dx} G(\bar{h}) \ , \qquad (2\ 25)$$

which is a constant as can be seen from the equation of continuity

From equations(2 24) and (2 25) we have the equation determining the pressure as follows:

$$\frac{d\bar{p}}{d\bar{x}} = \frac{\bar{h}}{G(\bar{h})} - \frac{\bar{Q}}{G(\bar{h})} \ , \qquad (2\ 26)$$

where $\bar{Q} = \frac{2Q}{Uh_o}$, $\bar{p} = \frac{h_o^2 p}{3UL(2\mu+\mu_1)}$, $\bar{x} = \frac{x}{L}$

and $\bar{h} = \frac{h}{h_o}$

Integrating equation (2 26) and using the conditions,

$$\begin{aligned} \bar{p} &= 0 \quad \text{at} \quad \bar{x} = 0 \\ \bar{p} &= 0 \quad \text{at} \quad \bar{x} = 1 \end{aligned} \qquad (2\ 27)$$

we get the expressions for pressure and flow flux as follows

$$p = \int_0^x \left[\frac{\bar{h}}{G(\bar{h})} \quad \frac{1}{G(h)} \quad \frac{\int_0^1 \frac{h}{G(h)}\,dx}{\int_0^1 \frac{1}{G(h)}\,dx}\right] dx \tag{2.28}$$

$$c\int_0^1 \frac{1}{G(h)}\,dx = \int_0^1 \frac{h}{G(h)}\,dx \tag{2.29}$$

The load capacity of the bearing (per unit width) is given by

$$W = \int_0^L p\,dx$$

which can be written as

$$\bar{W} = 6\int_0^1 p\,d\bar{x} = 6\int_0^1 x\,\frac{dp}{d\bar{x}}\,dx \tag{2.30}$$

where $\bar{W} = \dfrac{2Wh_o^2}{UL^2(2\mu+\mu_1)}$

In the following, the load capacity $\bar{W}$ is maximized with respect to the film thickness

$$\bar{h} \geq 1 \tag{2.31}$$

by using the approach of calculus of variation

2.3 METHOD-I VARIATIONAL APPROACH WITH FILM THICKNESS AS BOUNDED CONTROL VARIABLE

Following Maday [1968] and Shukla [1970] equations (2.26) and (2.31) are expressed as

$$\phi_1 = \frac{d\bar{p}}{d\bar{x}} - \frac{\bar{h}}{G(\bar{h})} + \frac{\bar{Q}}{G(\bar{h})} = 0 \ , \tag{2 32}$$

$$\phi_2 = \bar{h} - 1 - \psi^2 = 0 \ , \tag{2 33}$$

where ψ is a real function of $\bar{x}$ When $\psi = 0$, $\bar{h} = 1$ otherwise for non-zero ψ, $\bar{h} > 1$

It is required to find out the variabl s $\bar{p}$ and $\bar{h}$ such that $\bar{W}$ given by equation (2 30) attains a maximum

Let us define the augmented function F as follows

$$\Gamma = \Gamma p + \lambda_1(x)\ \phi_1(x) + \lambda_2(x)\ \phi_2(x) \ , \tag{2 34}$$

where λ's are undetermined Lagrange's multipliers, which are continuous in the interval $0 \le \bar{x} \le 1$, except possibly at a finite number of points

Using equations (2 32) and (2 33), the function Γ can be re-written as follows

$$\Gamma = 6\bar{p} + \lambda_1(\bar{x})\ \{\frac{dp}{dx} - \frac{\bar{h}}{G(\bar{h})} + \frac{Q}{G(\bar{h})}\} + \lambda_2(x)\ \{h - 1 - \psi^2\} \tag{2 35}$$

Since $\frac{d^2\bar{p}}{d\bar{x}^2}$ and $\frac{d\bar{h}}{d\bar{x}}$ do not occur in the governing equations (2 32) to (2 35), $\frac{d\bar{p}}{d\bar{x}}$ and $\bar{h}$ are needed to be only piecewise continuous in the interval $0 \le \bar{x} \le 1$ Following Leitmann [1962] , the Euler-Lagranges equations can be written from equation (2 35) as follows

$$6 = \frac{d\,\lambda_1}{dx} \quad , \tag{2 36}$$

$$\frac{\lambda_1}{G^2}\;[\bar{Q}\,G' + G - h\,G'] + \lambda_2 = 0 \quad , \tag{2 37}$$

$$\lambda_2\,\psi = 0 \;, \tag{2 38}$$

where dash denotes differentiation with respect to $\bar{h}$ As the augmented function F does not involve $\bar{x}$ explicitely, we get the following fundamental relation from equation (2 35)

$$6\bar{p} \quad \lambda_1 \frac{dp}{d\bar{x}} = C \quad , \tag{2 39}$$

where C is an integration constant.

The Weierstrass-Erdmann conditions, which apply at a corner of the extremal arc where $\frac{d\bar{p}}{d\bar{x}}$ and $\bar{h}$ are discontinuous, give

$$\lambda_1\Big|_{x=x_0\ 0} = \lambda_1\Big|_{x=\bar{x}_0+0} \tag{2 40}$$

$$\left(6p - \lambda_1 \frac{dp}{dx}\right)\Big|_{\bar{x}=x_0\ 0} = \left(6p - \lambda_1 \frac{d\bar{p}}{dx}\right)\Big|_{\bar{x}=x_0+0} \tag{2 41}$$

where $\bar{x} = \bar{x}_0$ is a point of discontinuity for the extremal and $\bar{x}_0 - 0$, $\bar{x}_0 + 0$ denote conditions immediately before and after $\bar{x}_0$ respectively

A necessary condition for the existence of local maximum for the load capacity $\bar{W}$ is given by Weierstrass E-function, Leitmann [1962], which is equivalent to the requirement that

$$E = \lambda_1 \frac{(\bar{h} - \bar{Q})}{G} \tag{2 42}$$

must be minimum with respect to $\bar{h}$

From equations (2 40) and (2 41) we see that λ_1 and $6\bar{p} - \lambda_1 \frac{d\bar{p}}{d\bar{x}}$ are continuous at the corner $\bar{x} = \bar{x}_o$ where $\frac{d\bar{p}}{d\bar{x}}$ and $\bar{h}$ are discontinuous Since $\bar{p}$ is continuous at $\bar{x} = \bar{x}_o$, from equations (2 40) and (2 41) we must have $\lambda_1(\bar{x}_o) = 0$ Then integrating equation (2 36) we get

$$\lambda_1(\bar{x}) = 6(\bar{x} - \bar{x}_o) \tag{2 43}$$

It is clear from equation (2 43) that $\lambda_1 \leq 0$ in $0 \leq \bar{x} \leq \bar{x}_o$ and $\lambda_1 \geq 0$ in $\bar{x}_o \leq \bar{x} \leq 1$ Also from equation (2 37) we have $\lambda_2 = 0$ at $\bar{x} = \bar{x}_o$ Now from equation (2 38) we have either $\lambda_2 = 0$ or $\psi = 0$ or both are zero From equation (2 33) it can be seen that the condition $\psi = 0$ implies that $\bar{h} = 1$ for both zero and non-zero λ_2 and the condition $\psi \neq 0$ gives $\bar{h} > 1$ and $\lambda_2 = 0$ Now for $\lambda_2 = 0$ and $\lambda_1 \neq 0$, we have from equation (2 37)

$$\bar{Q} = \bar{h}_m - \frac{G(\bar{h}_m)}{G'(\bar{h}_m)}, \tag{2 44}$$

where $\bar{h}_m$ is the optimum value of $\bar{h}$ in this region On substituting this value of $\bar{Q}$ in equation (2 29) it can be noted that $\bar{h}_m \neq 1$ Hence $\lambda_2 = 0$ and $\lambda_1 \neq 0$ imply that $\psi \neq 0$ Again if $\lambda_2 \neq 0$, ψ must be zero

in order that the equation (2 38) may be satisfied, and this corresponds to the region where $\bar{h} = 1$ This suggests that $\bar{h}$ is step function and it changes its value where λ_1 changes sign (i e at $\bar{x} = \bar{x}_0$) It now suffices to take the following profile for $\bar{h}$ corresponding to maximum of $\bar{W}$

$$\begin{aligned} \bar{h} &= \bar{h}_m , \quad 0 \le \bar{x} \le \bar{x}_0 , \\ \bar{h} &= 1 \quad , \quad \bar{x}_0 \le \bar{x} \le 1 , \end{aligned} \tag{2 45}$$

where $\bar{h}_m$ is given by equation (2 44) The qualitative behaviour of λ_1, λ_2 and $\bar{h}$ can be seen from figure no (2 3)

It can also be noted from equation (2 42) that for maximum of $\bar{W}$, the condition $\frac{dL}{d\bar{h}} = 0$ is equivalent to equation (2 44)

Using the profile given by (2 45) in equation (2 29) the flow flux of the lubricant is given by

$$\bar{Q} = \frac{\bar{h}_m \bar{x}_0 G(1) + (1 - \bar{x}_0) \ G(\bar{h}_m)}{\bar{x}_0 G(1) + (1 - \bar{x}_0) \ G(\bar{h}_m)} \tag{2 46}$$

Similarly the load capacity is given by using equations (2 30), (2 44), (2 45) and (2 46) as

$$\bar{W}_m = \frac{3(1 + \frac{\bar{\mu}_1}{2}) \ \bar{x}_0 (1 - \bar{x}_0) \ (\bar{h}_m - 1)}{\bar{x}_0 G(1) + (1 - \bar{x}_0) \ G(\bar{h}_m)} , \tag{2 47}$$

where $\bar{W}_m = (1 + \frac{\bar{\mu}_1}{2}) \ \bar{W}$, $\bar{W}_m = \frac{Wh_0^2}{\mu U L^2}$ and $\bar{\mu}_1 = \frac{\mu_1}{\mu}$ Here $\bar{x}_0$ is still undetermined, and if we maximize $\bar{W}_m$ with respect to $\bar{x}_0$ as well, then from the condition $\frac{\partial \bar{W}_m}{\partial \bar{x}_0} = 0$, we obtain

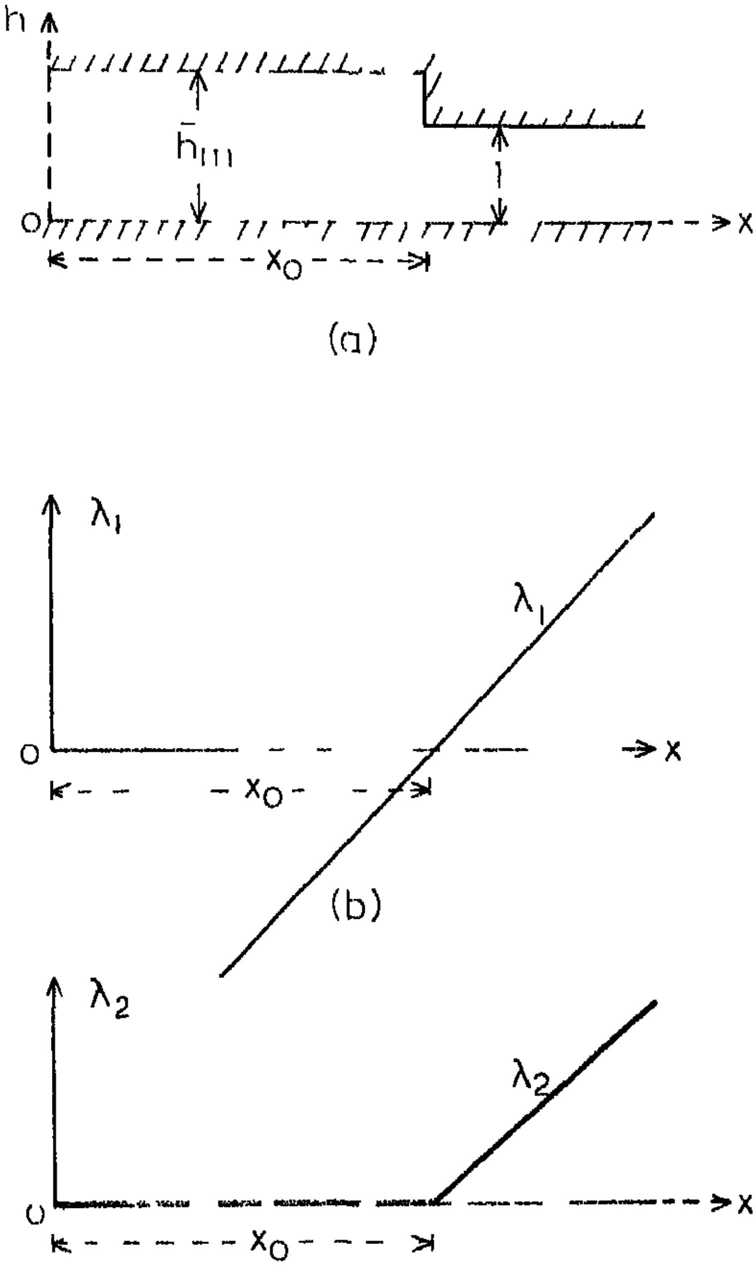


FIG. 2.3 (a) OPTIMUM PROFILE FOR FILM THICKNESS RATIO
(b) QUALITATIVE BEHAVIOUR OF λ_1
(c) QUALITATIVE BEHAVIOUR OF λ_2

$$\left(\frac{\bar{x}_o}{1-\bar{x}_o}\right)^2 = \frac{G(\bar{h}_m)}{G(1)} \tag{2 48}$$

This equation gives the step location for maximum load Further from equations (2 44) and (2 46), on eliminating $\bar{Q}$, we have

$$\frac{\bar{x}_o}{1-\bar{x}_o} = \frac{(\bar{h}_m - 1)\, G'(\bar{h}_m) - G(\bar{h}_m)}{G(1)} \tag{2 49}$$

The equations (2 48) and (2 49) determine $\bar{x}_o$ and $\bar{h}_m$ for maximum $\bar{W}_m$ Eliminating $\bar{x}_o$ from these equations we obtain the final equation determining $\bar{h}_m$ as

$$G(1)\, G(\bar{h}_m) - \{(\bar{h}_m - 1)\, G'(\bar{h}_m) - G(\bar{h}_m)\}^2 = 0 \tag{2 50}$$

The equation (2 50) is solved numerically for $\bar{h}_m$ for different M_1 and $\bar{\mu}_1$ and then the corresponding $\bar{x}_o$ is obtained from equation (2 48) The pair of $\bar{x}_o$ and $\bar{h}_m$ when substituted in equation (2 47) gives maximum $\bar{W}_m$ The force of friction F_m at the slider (per unit width) is given by

$$- F_m = \int_0^{x_o L} (\mu+\mu_1)\left(\frac{\partial u}{\partial y}\right)_{y=0} dx + \int_{\bar{x}_o L}^{L} (\mu+\mu_1)\left(\frac{\partial u}{\partial y}\right)_{y=0} dx ,$$

which on using equations (2 13) [with $u_1 = u$], (2 26) and (2 45) gives

$$-\Gamma_m = \frac{F_m h_o}{\mu\, UL} = \left(1 + \frac{\bar{\mu}_1}{2}\right) \{\bar{x}_o F_1 + (1 \quad x_o) F_2\} \tag{2 51}$$

where $\bar{F}_1 = \frac{3\bar{h}_m}{G(\bar{h}_m)} - \frac{2\alpha_1}{2\alpha_1 \bar{h}_m + \frac{\mu_1}{1+\mu_1} \tanh(\alpha_1 \bar{h}_m)}$

and $\bar{F}_2 = \frac{3}{G(1)} \{(\bar{h}_m - 1) \frac{G(\bar{h}_m)}{G'(\bar{h}_m)}\} - \frac{2\alpha_1}{2\alpha_1 + \frac{\mu_1}{1+\mu_1} \tanh \alpha_1}$

Also, the dimensionless coefficient of friction is given by

$$c_f = \frac{\bar{F}_m}{\bar{W}_m} \qquad (2.52)$$

2.4 METHOD-II MAXIMIZATION OF THE FUNCTIONAL $\bar{W}$

In this section the functional $\bar{W}$ is to be maximized with respect to the film thickness ratio $\bar{h} \geq 1$ by following the techniques of calculus of variation, Rayleigh [1918]

From equations (2.26) or (2.28) and (2.30) the functional $\bar{W}$ can be re-written as,

$$\bar{W} = 6[\int_0^1 \frac{\bar{Q}\,\bar{x}}{G(\bar{h})} dx - \int_0^1 \frac{\bar{h}\,\bar{x}}{C(\bar{h})} dx] \qquad (2.53)$$

where $\bar{Q}$ is given by the equation (2.29)

For an admissible variation $\delta\bar{h}$ in $\bar{h}$ there would be a corresponding change $\delta\bar{W}$ in $\bar{W}$, the expression for which can be written from equation (2.53) as

$$\delta W = \int_0^1 \frac{6}{G^2(h)} \phi(\bar{h}) \left\{\bar{x} - \frac{\int_0^1 \frac{\bar{x}\, d\bar{x}}{G(h)}}{\int_0^1 \frac{dx}{G(h)}}\right\} \delta\bar{h}\, dx \,, \tag{2 54}$$

where

$$\phi(\bar{h}) = \bar{h}G'(\bar{h}) - G(\bar{h}) - \bar{Q}G'(\bar{h}) \tag{2 55}$$

For maximum $\bar{W}$ the profile for $\bar{h}(\bar{x})$ should be such as to make $\delta\bar{W}$ always negative or zero It is se n from equation (2 54) that the sign of $\delta\bar{W}$ is given by the product of $\phi(\bar{h})$, $x - \dfrac{\int_0^1 \frac{\bar{x}d\bar{x}}{G(\bar{h})}}{\int_0^1 \frac{d\bar{x}}{G(\bar{h})}}$ and $\delta\bar{h}$

To make $\delta\bar{W}$ negative or zero, it suffices to choose the following profile for $\bar{h}(\bar{x})$

$$\bar{h} = \bar{h}_m \,, \quad 0 \leq \bar{x} \leq \bar{x}_0 \,,$$
$$\bar{h} = 1 \,, \quad \bar{x}_0 \leq \bar{x} \leq 1 \,, \tag{2 56}$$

such that

$$\phi(\bar{h}_n) = \bar{h}_m G'(\bar{h}_m) - G(\bar{h}_m) - \bar{Q}\, G'(\bar{h}_m) = 0 \tag{2 55)'$$

at the upper step $0 \leq \bar{x} \leq \bar{x}_0$ and $\bar{x}_0$ is so chosen as to make the following expression positive i e

$$x_0 \int_0^1 \frac{dx}{G(\bar{h})} - \int_0^1 \frac{x\, dx}{G(\bar{h})} \geq 0 \quad \text{for } \bar{x} \geq x_0 \tag{2 57}$$

The behaviour of $\phi(\bar{h})$, given by equation (2 55),with respect to $\bar{h}$

is shown in figure no (2 4) It is noted from this graph that $\phi(\bar{h})$ vanishes for $\bar{h} = \bar{h}_m$ and it is negative when $\bar{h} < \bar{h}_m, (\bar{x}_o \leq \bar{x} \leq 1)$ and positive for $\bar{h} > \bar{h}_m, (\bar{h}_m > 1)$ Thus for the region $0 \leq \bar{x} \leq \bar{x}_o$, $\bar{h} = \bar{h}_m$ and $\phi(\bar{h}_m) = 0$ Therefore, $\delta\bar{W}$ vanishes whatever may be the product of the other two terms But in the region $\bar{x}_o \leq \bar{x} \leq 1$, $\bar{h} = 1$, $\phi(\bar{h})$ is negative and $\delta\bar{h}$ is positive and therefore $\delta\bar{W}$ is negative because of the inequality (2 57)

On using the optimum profile in the inequality (2 57) we get the condition determining $\bar{x}_o$ as follows:

$$\frac{x_o^2}{(1-\bar{x}_o)^2} \geq \frac{G(\bar{h}_m)}{G(1)} \tag{2 58}$$

Similarly from the equation (2 55)' the step height ratio $\bar{h}_m$ for optimum $\bar{W}$ is given by

$$\bar{Q} = \bar{h}_m - \frac{G(\bar{h}_m)}{G'(\bar{h}_m)} \tag{2 59}$$

Since the conditions (2 58) and (2 59) ar the same as equations (2 48) and (2 44) respectively, obtained in the previous Section 2 3, therefore this method of optimization of the functional $\bar{W}$ would give the same results for various characteristics of the slider bearing as already obtained in the previous section

2 5 RESULTS AND DISCUSSIONS

The variations of $\bar{Q}$ and $\bar{W}_m$ are shown respectively in figures (2 5) and (2 6) It is seen that $\bar{Q}$ decreases and $\bar{W}_m$

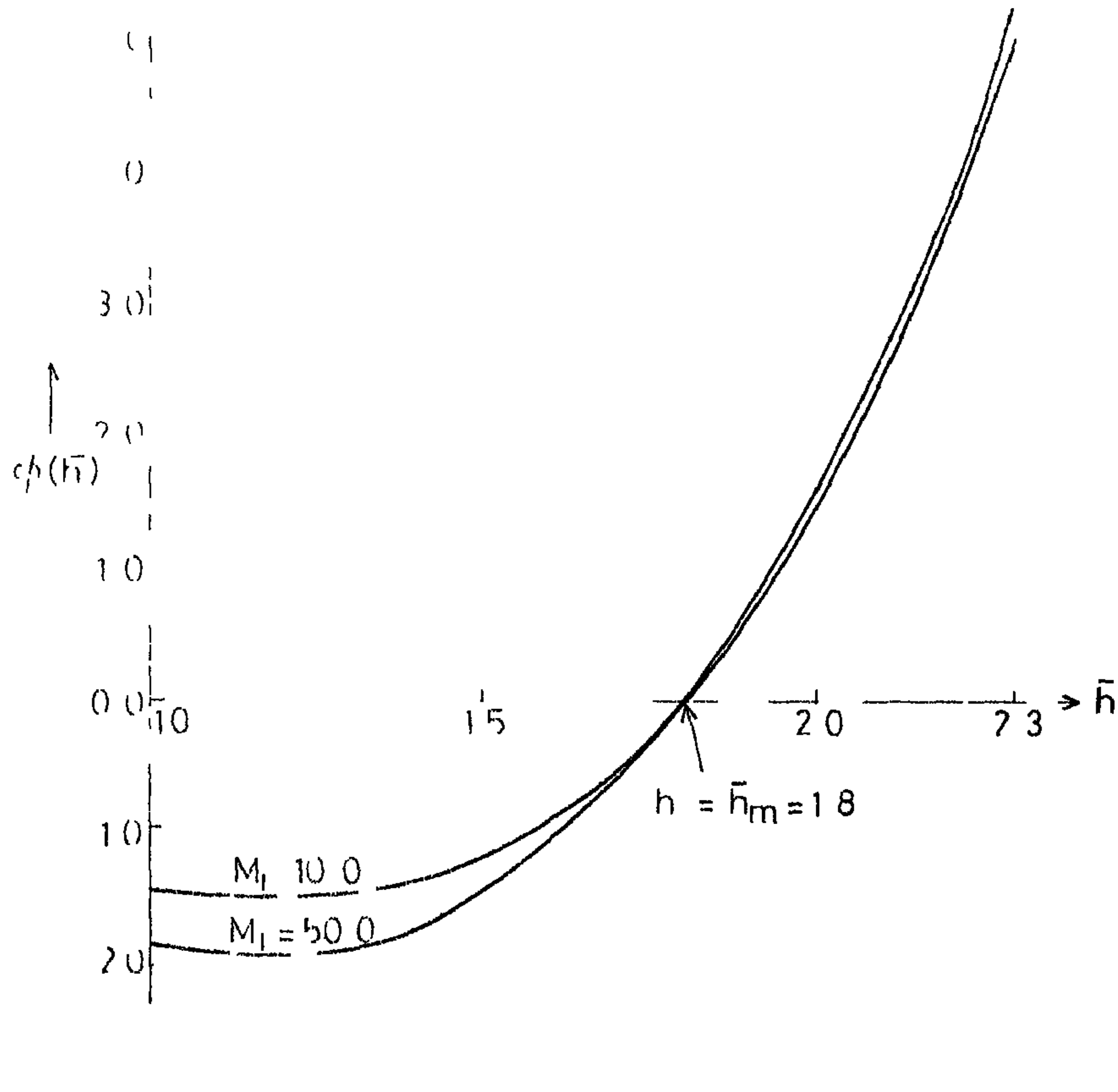


FIG 2 4 VARIATION OF $\phi(\bar{h})$ WITH $\bar{h}$ FOR DIFFERENT M_1 AND $\bar{\mu}_1 = 0\ 8$ $\bar{h}_m = 1\ 8$

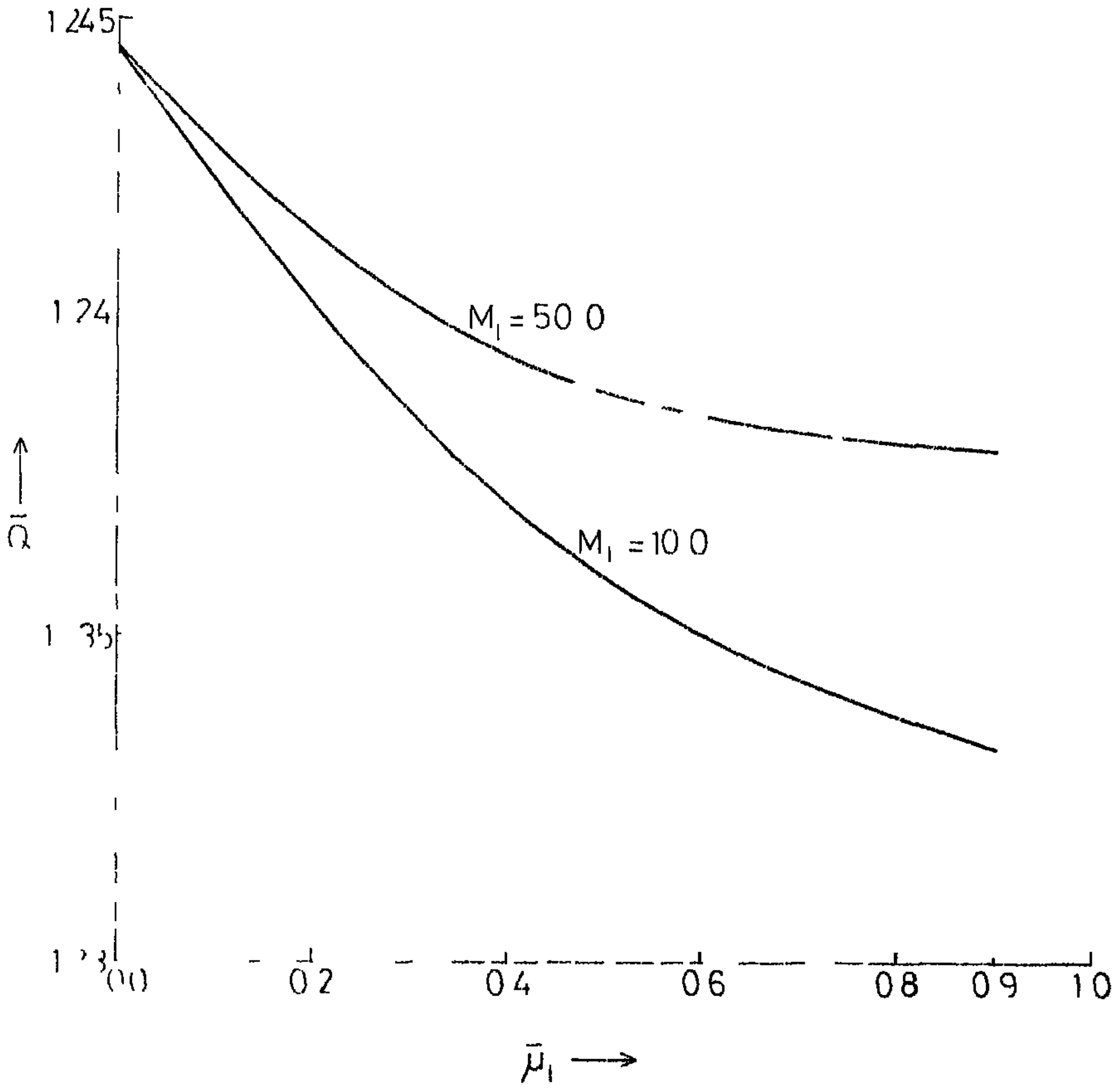


FIG 2 5 VARIATION OF $\bar{Q}$ WITH $\bar{\mu}_1$ FOR DIFFERENT M_1

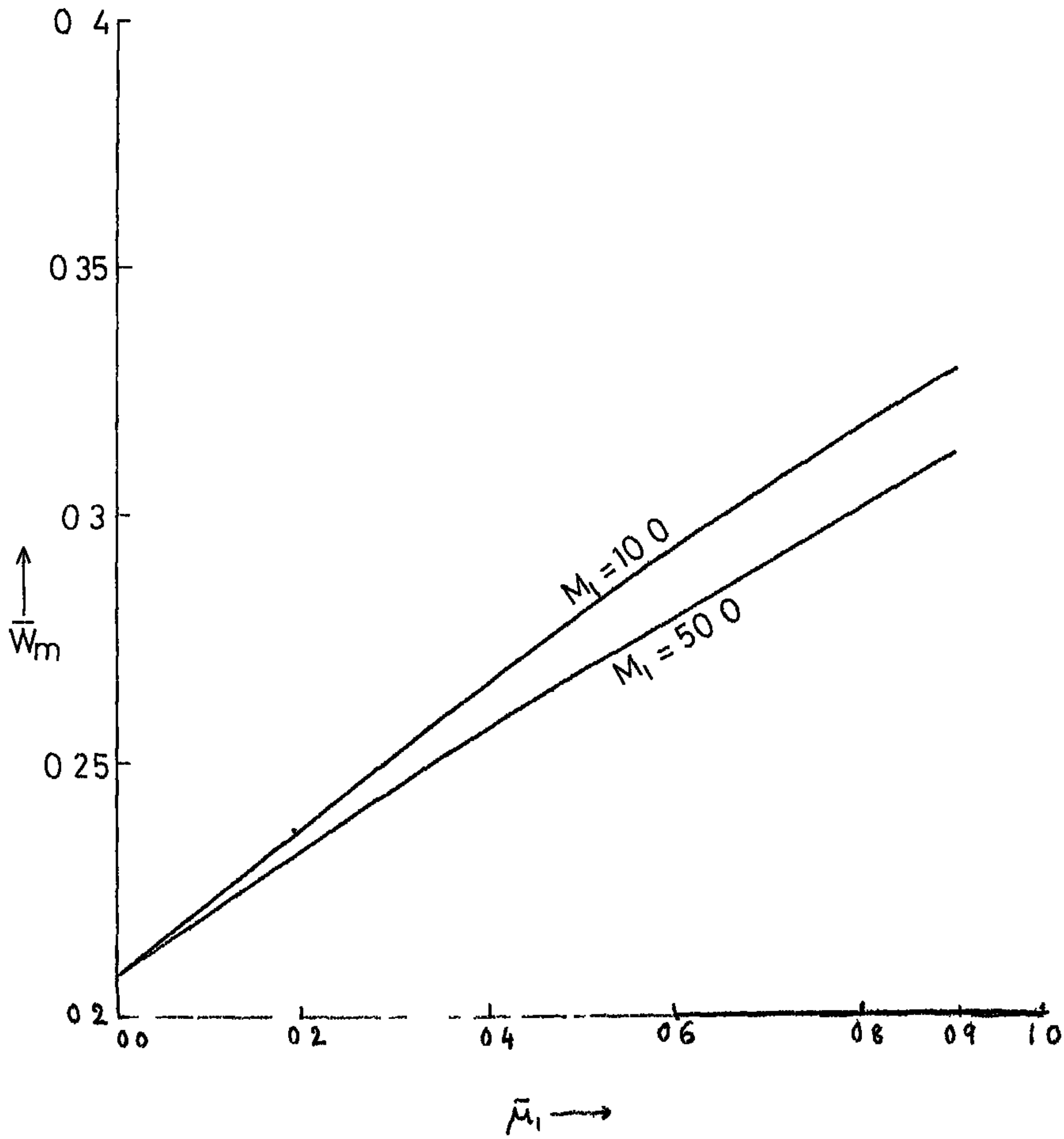


FIG 2 6 VARIATION OF $\bar{W}_m$ WITH $\bar{\mu}_1$ FOR DIFFERENT M_1

increases as $\bar{\mu}_1$ increases It is further seen that $\bar{Q}$ increases and $\bar{W}_m$ decreases as M_1 increases for various values of μ_1 The corresponding values of flux and load capacity for the classical Newtonian case are obtained by letting $\mu_1 \to 0$ or $M_1 \to \infty$

The variation of $\bar{F}_m$ is manifested through figure no (2 7) Here also it is seen that $\bar{F}_m$ increases as μ_1 increases The variation of the dimensionless coefficient of friction c_f is shown in figure no (2 8) It is noted that c_f decreases as μ_1 increases and thus the increase in f iction force is more than compensated by increase in load capacity

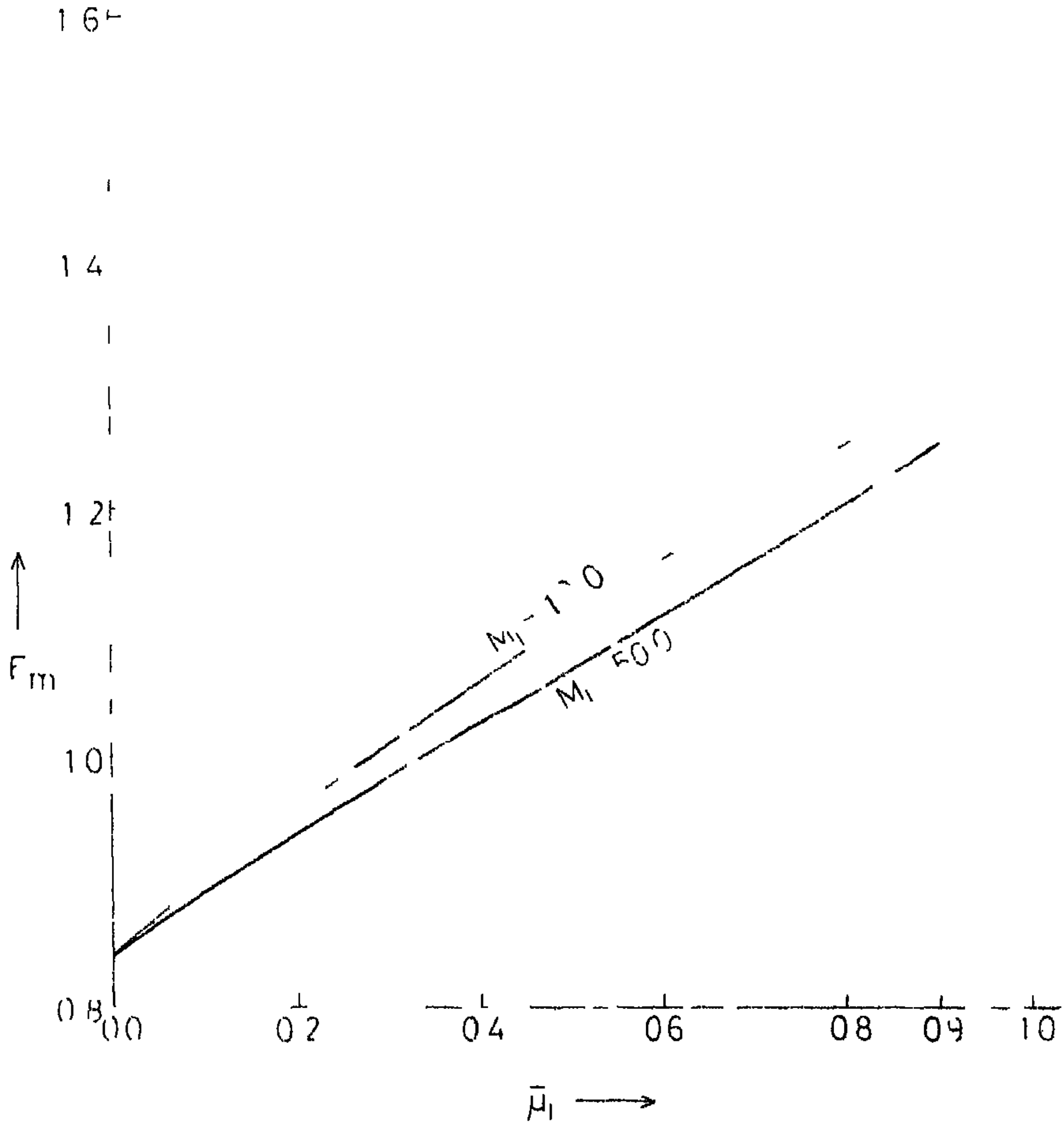


FIG 2.7 VARIATION OF $\bar{F}_m$ WITH μ_1 FOR DIFFERENT M_1

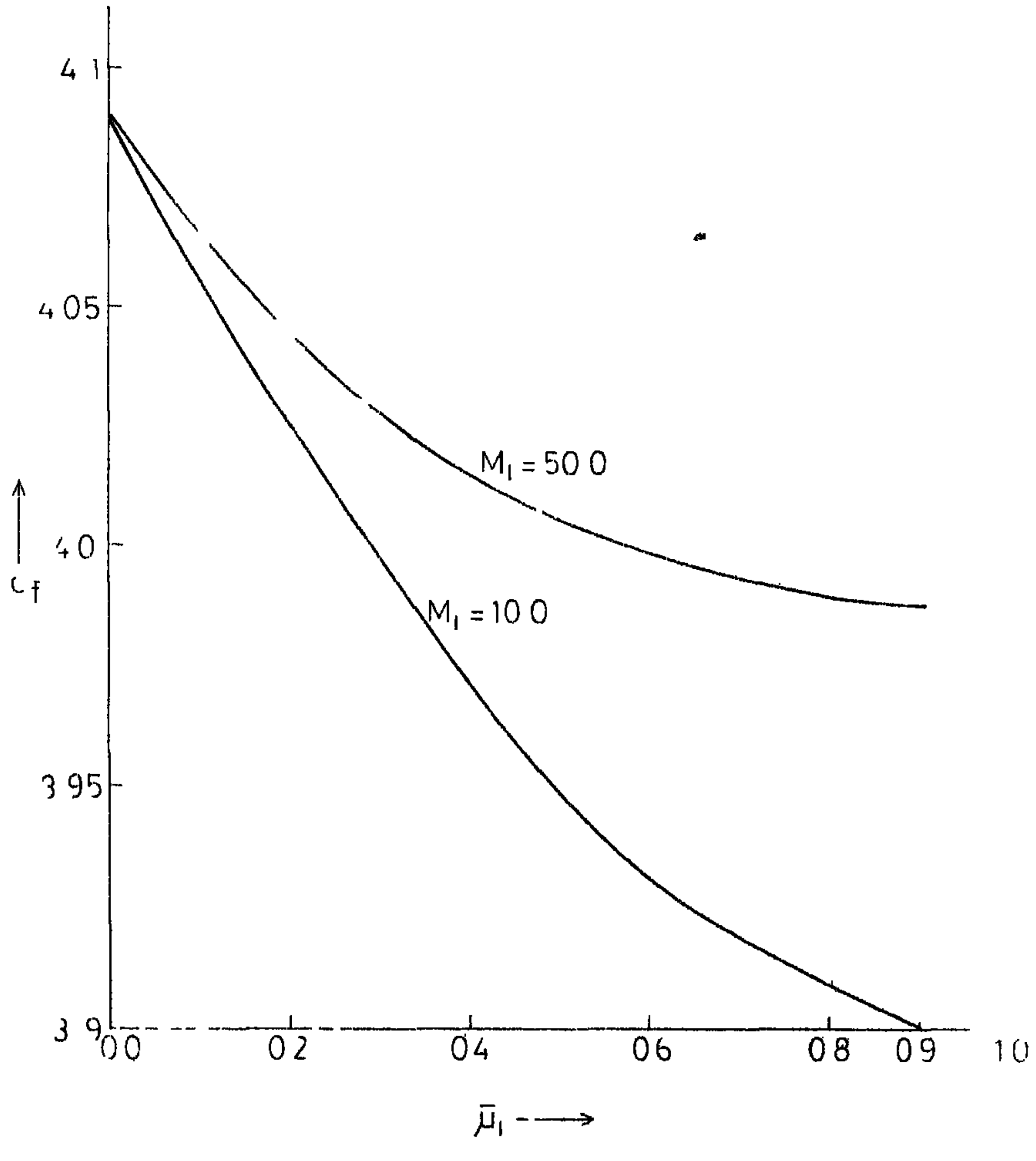


FIG 2 8 VARIATION OF c_f WITH $\bar{\mu}_1$ FOR DIFFERENT M_1

CHAPTER - III

EXTERNALLY PRESSURISED OPTIMUM BEARING WITH MICROPOLAR FLUID AS A LUBRICANT

In Chapter II, we have studied the characteristics of one dimensional optimum slider bearing by characterizing the lubricant suspension as a micropolar fluid In this chapter also, we study the behaviour of the lubricant suspension containing additives in the case of an externally pressurised bearing by considering the same model The approach of calculus of variation is applied to study the optimum film thickness profile for the maximum load capacity of the bearing, Walker and Osterle [1961]

3 1 BASIC EQUATIONS

Consider the flow of a lubricant suspension, characterised as micropolar fluid, in an externally pressurised circular bearing Because of externally applied supply pressure, the suspension flows only in the radial direction and there is no component of the velocity in the θ-direction due to symmetry The micro-rotation vector is assumed to have only θ-component, the z-component being negligibly small because of small film thickness in comparison to the other dimensions of the bearing The geometrical configuration is illustrated in figure no (3 1) Thus by considering

$$\vec{V} = (u, 0, 0)$$
$$\vec{\omega} = (0, \omega, 0) \qquad (3\ 1)$$

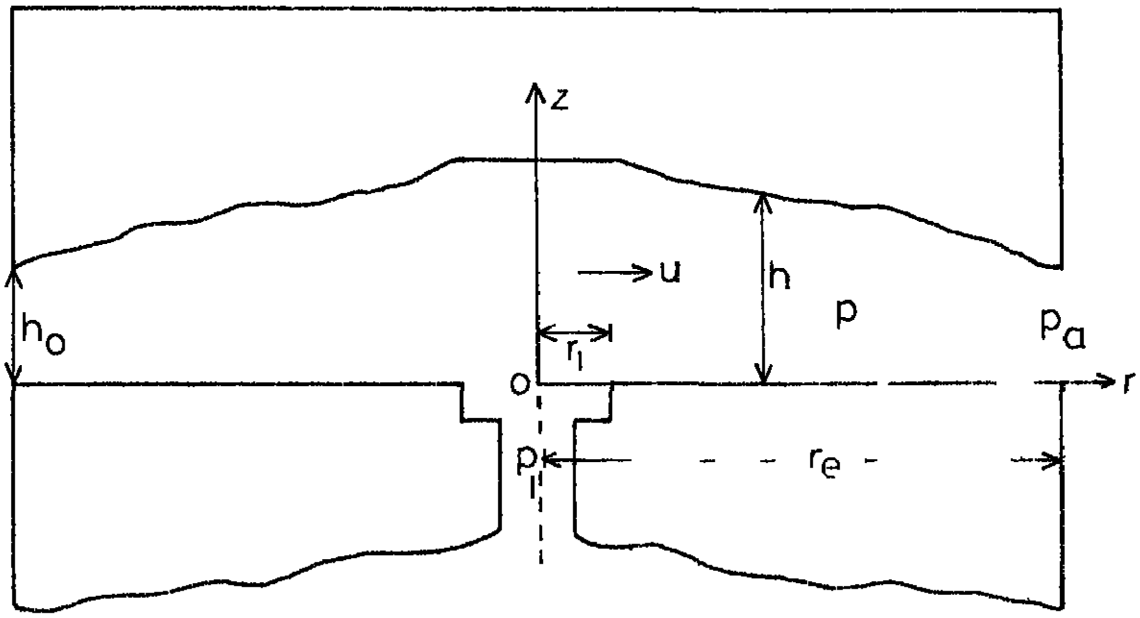


FIG 3 1 EXTERNALLY PRESSURISED BEARING WITH MICROPOLAR LUBRICANT

and using the usual lubrication assumptions in equations (2 1), (2 2) and (2 3) we get the following equations governing the flow of the lubricant:

$$(\mu + \mu_1) \frac{\partial^2 u}{\partial z^2} \quad \mu_1 \frac{\partial \omega}{\partial z} - \frac{dp}{dr} = 0 \tag{3 2}$$

$$\gamma \frac{\partial^2 \omega}{\partial z^2} + \mu_1 \frac{\partial u}{\partial z} \quad 2\mu_1 \, \omega = 0 \tag{3 3}$$

The boundary conditions for u and ω at the surfaces are

$$u = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = h \ , \tag{3 4}$$

$$\omega = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = h \tag{3 5}$$

Solving equations(3 2) and (3 3) and using the conditions (3 4) and (3 5) we get the expressions for u and ω as follows

$$u = A[\beta \sinh \lambda z + 2z] + B\beta(\cosh \lambda z \quad 1) + \frac{z^2}{2\mu+\mu_1} \frac{dp}{dr} \ , \tag{3 6}$$

$$\omega = A(\cosh \lambda z - 1) + B \sinh \lambda z - \frac{z}{2\mu+\mu_1} \frac{dp}{dr} \ , \tag{3 7}$$

where $A = \dfrac{h}{2(2\mu+\mu_1)} \dfrac{dp}{dr}$,

$$B = \frac{h}{2(2\mu+\mu_1)} \frac{dp}{dr} \quad \frac{1 + \cosh \lambda h}{\sinh \lambda h} \ ,$$

$$\beta = \frac{\lambda \gamma}{\mu_1} \quad \frac{2}{\lambda} \ ,$$

$$\lambda^2 = \frac{(2\mu+\mu_1) \ \mu_1}{(\mu+\mu_1)\gamma}$$

The flow flux is defined by

$$Q = \int_0^h 2\pi r u dy \tag{3 8}$$

which on using equation (3 6) gives

$$Q = -\frac{\pi h_o^3}{3(2\mu+\mu_1)} r \frac{dp}{dr} \; C(\bar{h}) \; , \tag{3 9}$$

where

$$C(\bar{h}) = C(\frac{1}{h_o}) = \bar{h}^3 + \frac{3\mu_1 h}{2(1+\mu_1)} \; \frac{1 \quad \alpha_1\bar{h} \coth \alpha_1\bar{h}}{\alpha_1^2} \; ,$$

$$\bar{h} = \frac{h}{h_o} \; , \; \bar{\mu}_1 = \frac{\mu_1}{\mu} \quad \alpha_1^2 = \frac{\lambda^2 h_o^2}{4} = \frac{M_1 \bar{\mu}_1(2 + \mu_1)}{1 + \bar{\mu}_1}$$

$$M_1 = \frac{1}{4\bar{\gamma}} \; \& \; \bar{\gamma} = \gamma/\mu h_o^2$$

It can be seen from the equation of continuity (2 3) that Q is a constant

Integrating equation (3 9) and using the boundary conditions

$$\begin{aligned} p &= p_i \quad \text{at} \quad r = r_i \\ p &= p_a \quad \text{at} \quad r = r_e \end{aligned} \tag{3 10}$$

we get the final expressions for Q and p as

$$Q \int_1^p \frac{dx}{xC(\bar{h})} = \frac{\pi h_o^3}{3(2\mu+\mu_1)} (p_i - p_a) \tag{3 11}$$

$$p - p_a = (p_i - p_a)\left[1 - \frac{\int_1^x \frac{dx}{xG(\bar{h})}}{\int_1^R \frac{dx}{xG(\bar{h})}}\right] \qquad (3\ 12)$$

where $x = \dfrac{r}{r_i}$

The load capacity of the bearing is defined as

$$W = \pi(p_i - p_a)\, r_i^2 + \int_{r_i}^{r_e} 2\pi r (p - p_a)\, dr \qquad (3\ 13)$$

which on using equation (3 12) gives

$$\bar{W} = \frac{W}{\pi(p_i - p_a)\, r_i^2} = \frac{\int_1^R \frac{x}{G(\bar{h})}\, dx}{\int_1^R \frac{dx}{xG(\bar{h})}} \qquad (3\ 14)$$

where $R = \dfrac{r_e}{r_i} > 1$ and $p_i - p_a$ is prescribed

3 2 DETERMINATION OF OPTIMUM FILM THICKNESS FOR MAXIMUM LOAD CAPACITY

From equation (3 14), it can be noted that the load capacity is a function of $\bar{h}(x)$ In this section we determine the optimum profile for the film thickness $\bar{h}$ corresponding to maximum load capacity of the bearing

From equation (3 14) the load capacity can be re-written as

$$\bar{W} = \frac{g}{f} \;, \tag{3 15}$$

where
$$g = \int_1^R \frac{x\,dx}{G(\bar{h})} \tag{3 16}$$

and
$$f = \int_1^R \frac{dx}{xG(\bar{h})} \tag{3 16}$$

Now, any change $\delta\bar{h}$ in $\bar{h}$ would produce a corresponding change $\delta\bar{W}$ in $\bar{W}$ From equations (3 15) the expression for $\delta\bar{W}$ can be written as

$$\delta\bar{W} = \frac{f\delta g - g\delta f}{f^2} \tag{3 17}$$

which on using equation (3 16) gives

$$\delta\bar{W} = \frac{1}{f^2} \int_1^P F(x)\, \frac{G'(h)}{G^2(h)}\, \delta h \; dx \;, \tag{3 18}$$

where

$$F(x) = \frac{g}{x} - xf \tag{3 19}$$

Here we are interested in determining that profile for $\bar{h}(x)$ which would make $\delta\bar{W}$ negative for all possible variations in $\bar{h}$

It can be noted from the figures (3 2) and (3 3) that $G(\bar{h})$ and $G'(\bar{h})$ are positive and increase as $\bar{h}$ increases for $\bar{h} \geq 1$ Therefore from equation (3 18) it is clear that the sign of $\delta\bar{W}$ depends upon the product of $F(x)$ and $\delta\bar{h}$ It can be seen that $F(x)$ is

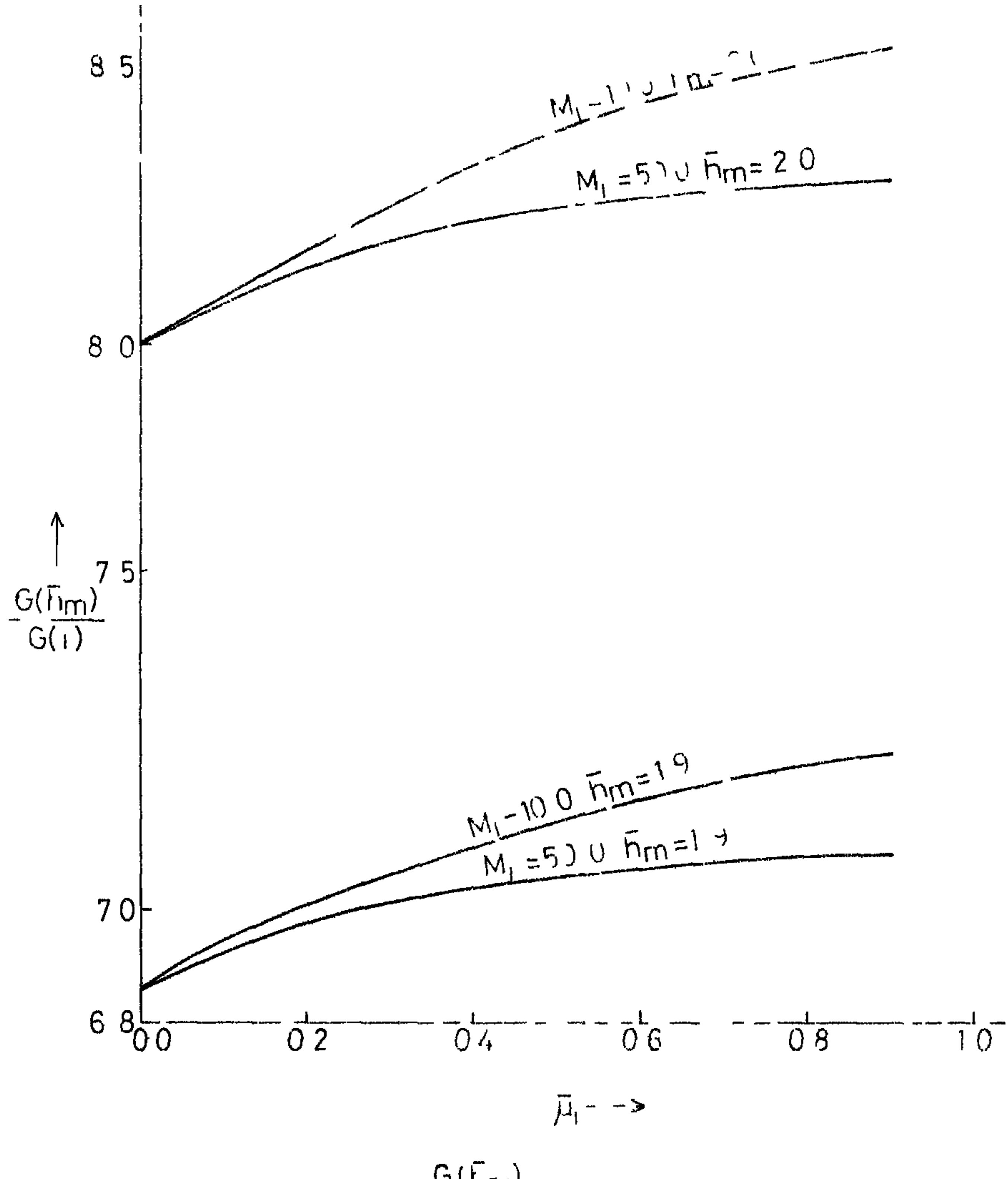


FIG 3 2 VARIATION OF $S_1 = \frac{G(\bar{h}_m)}{G(1)}$ WITH $\bar{\mu}_1$ FOR DIFFERENT M_1 AND $\bar{h}_m$

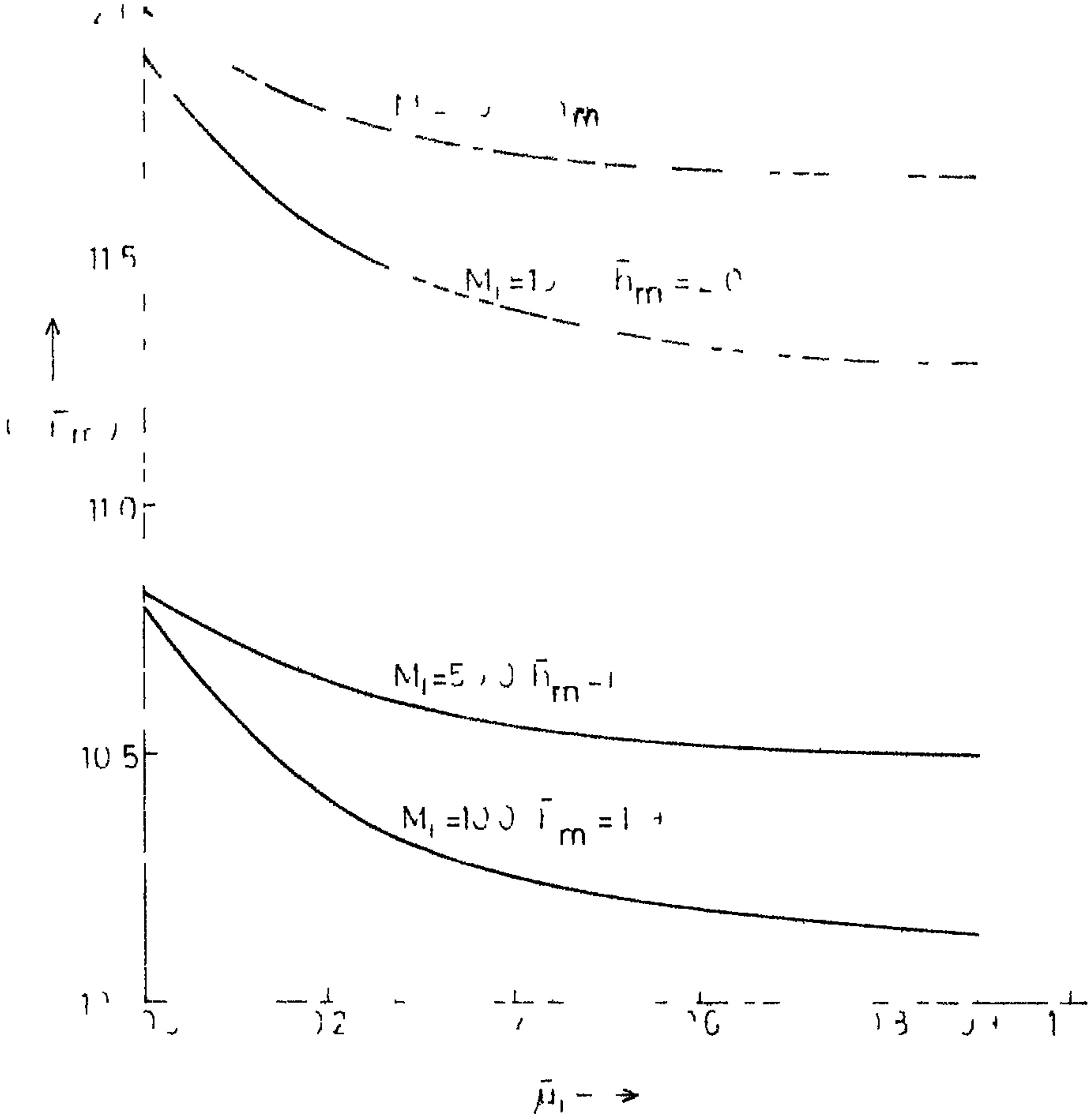


FIG ^ } VARIATION OF G($\bar{n}_m$) WITH $\bar{\mu}_l$ FOR DIFFERENT M_l AND $\bar{n}_m$

positive for values of x close to unity and negative for values approaching R, Rayleigh [1918] Suppose the value of x at which $F(x)$ changes sign is x_o, then from equation (3 19) we have

$$x_o^2 = \frac{g}{f} \tag{3 20}$$

The condition that $\delta \bar{W}$ is negative for any admissible variation $\delta \bar{h}$ would be satisfied if we choose the following profile for film thickness ratio,

$$\begin{aligned} \bar{h} &= \bar{h}_m \quad , \quad 1 \leq x \leq x_o \\ \bar{h} &= 1 \quad , \quad x_o \leq x \leq R \, , \end{aligned} \tag{3 21}$$

where $\bar{h}_m$ is the ratio of the maximum to the minimum allowable film thickness

It is easily noted that for $1 \leq x \leq x_o$, $\delta\bar{h}$ can only be negative and since $F(x)$ is positive in this range, $\delta\bar{W}$ is negative Again, for $x_o \leq x \leq R$, $\delta\bar{h}$ can only be positive and since $F(x)$ is positive in this range, $\delta\bar{W}$ is negative Thus, it is shown that $\delta\bar{W}$ is negative throughout the region $1 \leq x \leq R$ for all possible variations of $\bar{h}$ The load capacity for this type of film thickness profile can now be written by using equations (3 15) and (3 20) as

$$\bar{W}_m = x_o^2 \tag{3 22}$$

The relation between x_o and $\bar{h}_m$ can be obtained by using equations (3 20) and (3 21) as

$$x_o^2 = \frac{\int_1^{x_o} \frac{x\,dx}{G(\bar{h}_m)} + \int_{x_o}^{R} \frac{x\,dx}{G(1)}}{\int_1^{x_o} \frac{dx}{xG(\bar{h}_m)} + \int_{x_o}^{R} \frac{dx}{xG(1)}} , \tag{3 23}$$

which on integration and simplification gives

$$\frac{G(\bar{h}_m)}{G(1)} = \frac{x_o^2 \ln x_o - (x_o^2 - 1)}{(R^2 - x_o^2) - 2x_o^2 \ln \frac{R}{x_o}} = S_1 , \tag{3 24}$$

where $G(\bar{h}_m)$ and $G(1)$ are values of $G(\bar{h})$ when $\bar{h} = \bar{h}_m$ and $h = 1$ respectively

The equation (3 24) gives the relation between x_o, the step location and $\bar{h}_m$, the step height ratio, for maximum load capacity of the bearing The variations of $G(\bar{h}_m) / G(1)$ versus μ_1 are shown in figure no (3 2) It is seen that the function $S_1 = G(\bar{h}_m)/G(1)$ increases as $\bar{\mu}_1$ or $\bar{h}_m$ increases for fixed M_1 The variations of S_1 with $\frac{x_o}{R}$ are shown in figure no (3 4), from which it may be noted that the function S_1 increases as x_o increases for fixed R Since S_1 increases as μ_1 or $\bar{h}_m$ increases, it may be inferred that x_o increases as μ_1 or $\bar{h}_m$ increases Now from equation (3 22) it is noted that the maximum load capacity is proportional to x_o^2 and consequently this maximum load increases as μ_1 or $\bar{h}_m$ increases It is further noticed from figure no (3 2) that $G(\bar{h}_m) / G(1)$ decreases as M_1 increases, and so with the reasoning given as above it may be

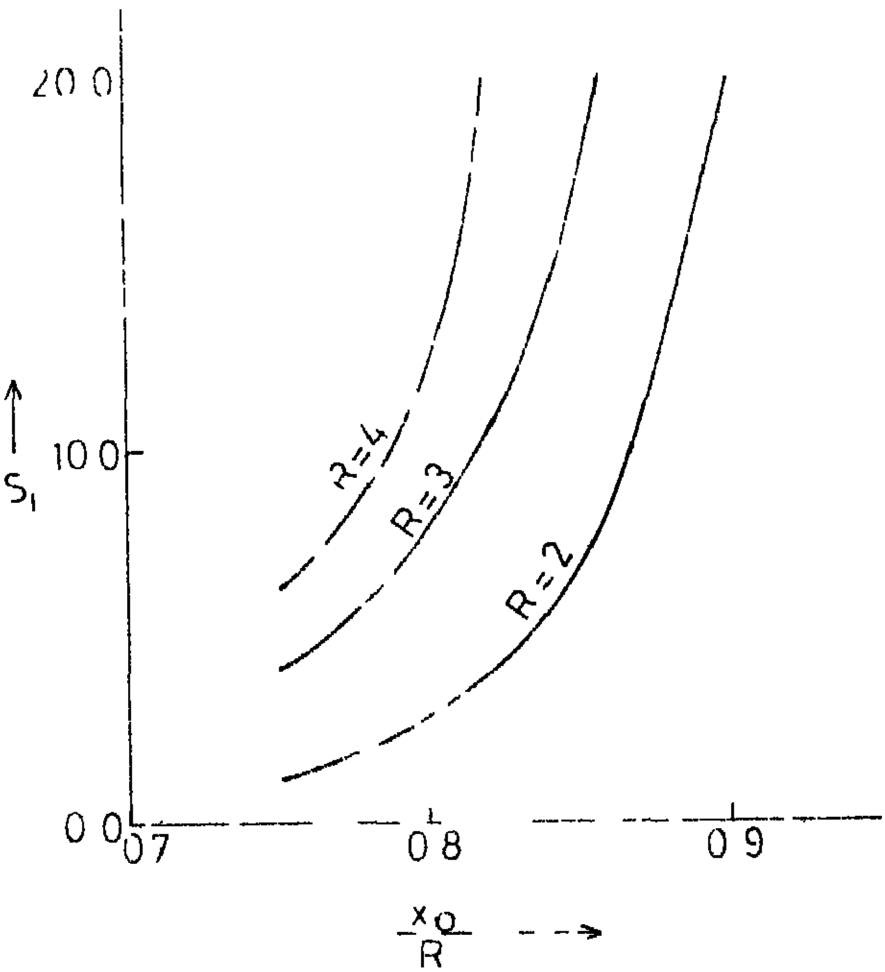


FIG 3 4 VARIATION OF S_1 WITH $\frac{x_0}{R}$ FOR DIFFERENT R

concluded that the maximum load capacity decreases as the parameter M_1 increases. It may also be noted that the behaviour of $\bar{W}_m$ tends to that of Newtonian lubricant when $\mu_1 \to 0$ or $M_1 \to \infty$. It is further remarked that since S_1 increases indefinitely as x_o approaches R, the optimum load capacity increases as the step position moves to the outer radius of the bearing.

From equation (3.11) the expression for the flow flux can be rewritten as

$$\bar{Q} = \frac{6\mu Q}{\pi h_o^3 (p_1 - p_a)} = \frac{1}{(1 + \frac{\bar{\mu}_1}{2})} \frac{1}{\int_1^R \frac{dx}{xG(\bar{h})}} \tag{3.25}$$

Making use of equations (3.21) in equation (3.25), the expression for $\bar{Q}$ can be written as follows

$$\bar{Q} = \frac{1}{(1 + \frac{\bar{\mu}_1}{2})} \frac{G(1)}{\ell n R - \{1 - \frac{G(1)}{G(\bar{h}_m)}\}\ell n x_o} \tag{3.26}$$

As pointed out earlier, x_o and $G(\bar{h}_m)$ increase as $\bar{h}_m$ increases and $G(\bar{h}_m) > G(1)$ for $\bar{h}_m > 1$ [see figure no. (3.2)]. Then from equation (3.26) it may be concluded that $\bar{Q}$ increases as $\bar{h}_m$ increases for fixed $\bar{\mu}_1$ and M_1.

Again, since $G(1)$ and $G(\bar{h}_m)$, $\bar{h}_m > 1$, decrease as $\bar{\mu}_1$ increases or M_1 decreases [see figures (3.5) and (3.6)], the integral on the right hand side of equation (3.25) increases as $\bar{\mu}_1$ increases or M_1 decreases. Hence it may be pointed out that $\bar{Q}$ decreases as $\bar{\mu}_1$ increases or M_1 decreases.

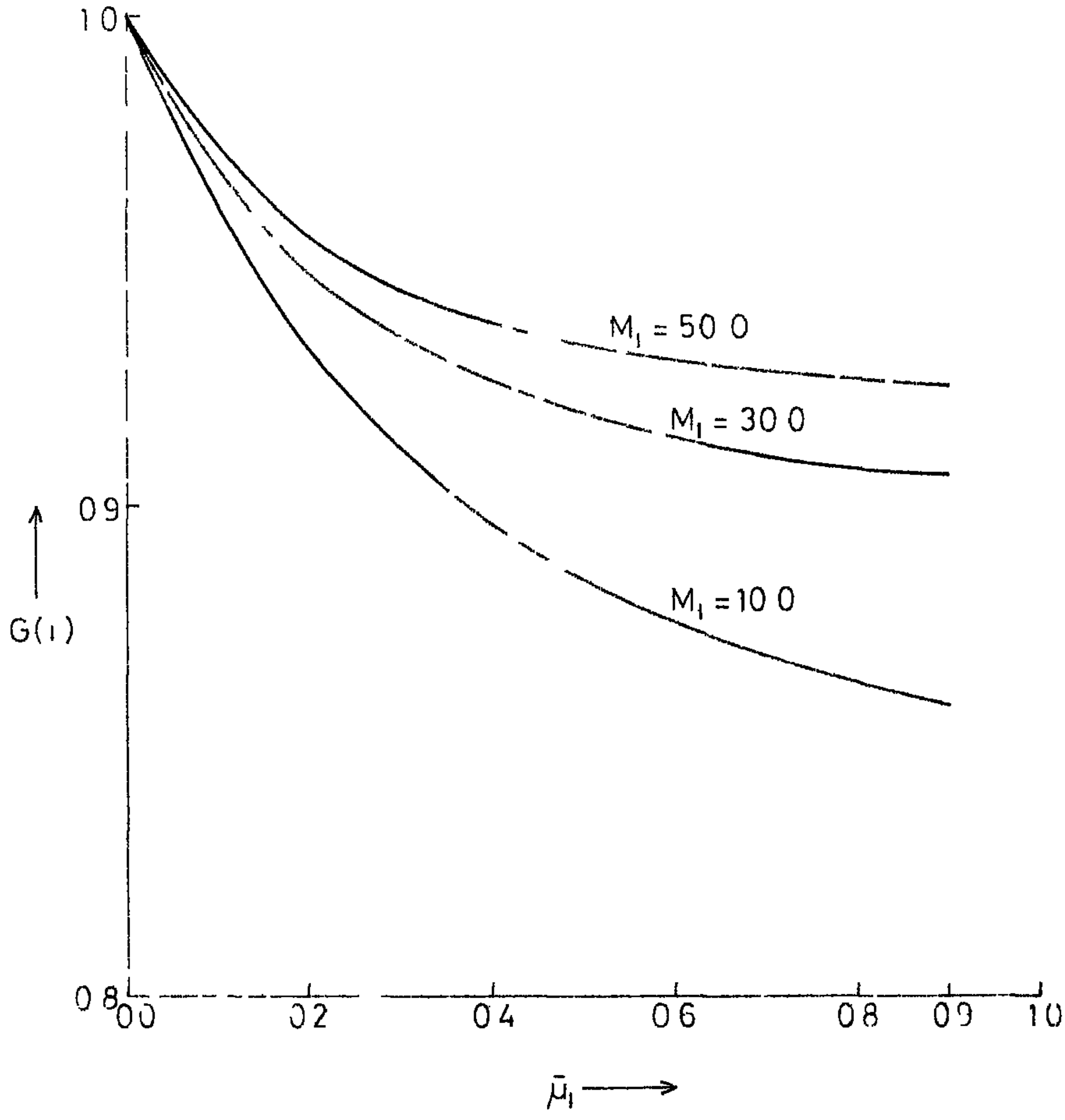


FIG 3 5 VARIATION OF G(1) WITH $\bar{\mu}_1$ FOR DIFFERENT M_1

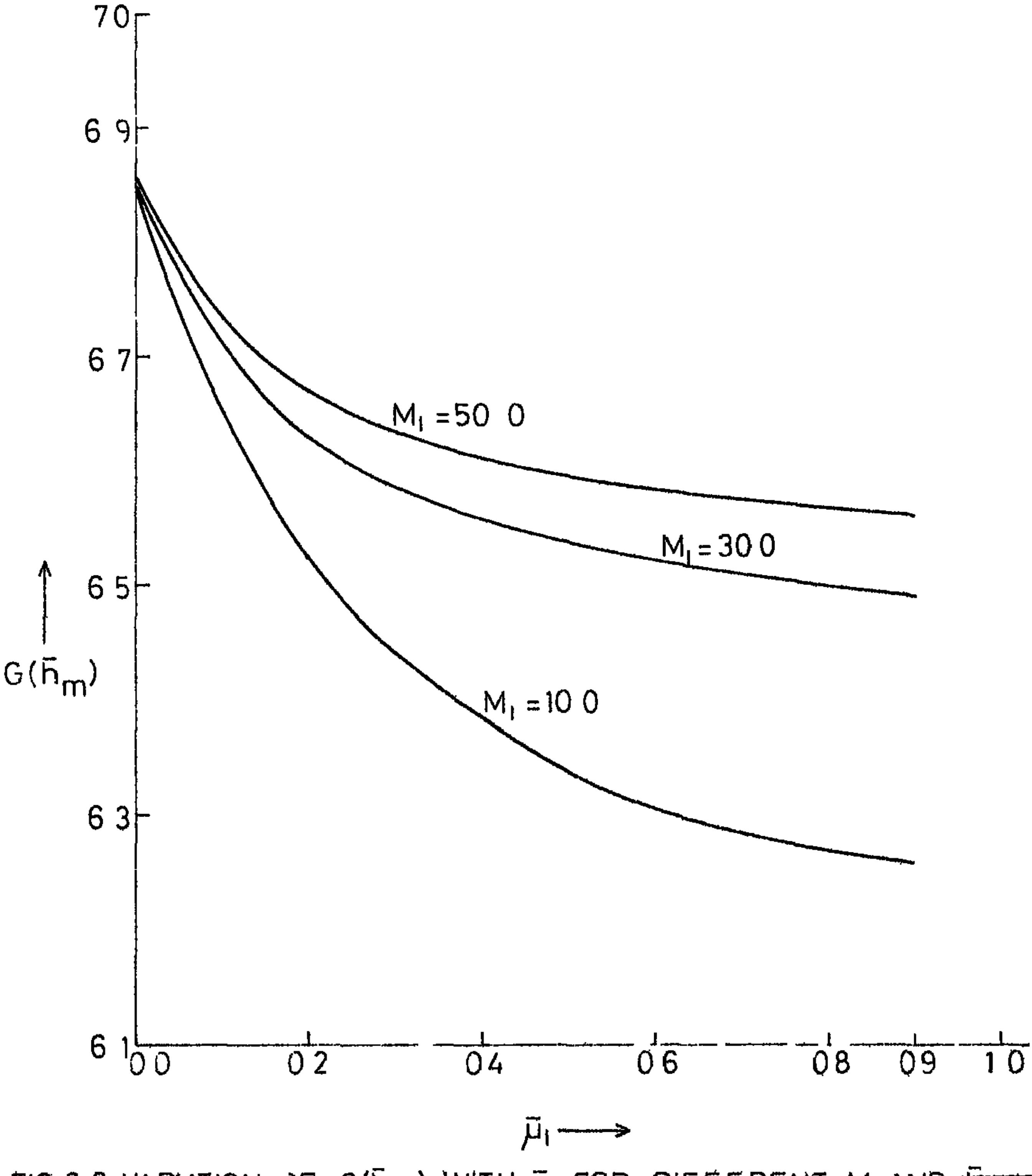


FIG 3 6 VARIATION OF $G(\bar{h}_m)$ WITH $\bar{\mu}_1$ FOR DIFFERENT M_1 AND $\bar{h}_m$ =

3 3 CONCLUSIONS

In this chapter, the characteristics of an externally pressurised optimum bearing with micropolar lubricant have been investigated Following conclusions may be drawn from the above analysis

(1) The maximum load capacity increases as the step height ratio $\bar{h}_m$, or the parameter μ_1 or both increase This maximum load capacity decreases as the parameter M_1 increases The Newtonian case is derived by letting $\bar{\mu}_1 \to 0$ or $M_1 \to \infty$

(2) For prescribed applied pressure, the flow flux increases as $\bar{h}_m$ increases and it decreases as $\bar{\mu}_1$ increases or as M_1 decreases

CHAPTER - IV

EFFECTS OF VISCOSITY VARIATION ON THE CHARACTERISTICS OF SLIDER AND HYDROSTATIC BEARINGS DUE TO CHANGE IN CONCENTRATION OF ADDITIVES

In the previous two chapters, we have studied the characteristics of slider and externally pressurised bearings with lubricants containing additives by considering them as micropolar fluids. In this chapter, we consider the effects of additives in lubricant through the change of viscosity of the base oil due to presence of additive in it.

It is well known that the viscosity of the lubricant changes with concentration of additives which in turn changes with the processes such as diffusion, convection, chemical reaction etc. present in the system, Braithwaite [1967]. As early as 1906, Einstein derived the following formula governing the viscosity of a dilute suspension of rigid spherical particles uniformly dispersed in a Newtonian liquid

$$\mu = \mu_o \left(1 + 2.5\, C_o\right) \tag{4.1}$$

Here C_o is the volume of the dispersed material per unit volume of the base oil. Since then various models have been proposed governing the viscosities of dispersions, Rutgers [1962] and emulsions, Sherman [1962]. In general, most of these models can be written as

$$\mu = \mu_o [1 + \lambda C_o + \lambda_1 C_o^2 + \lambda_2 C_o^3 + \ldots] \tag{4.2}$$

where λ's are parameters depending upon size, shape and nature of particles of the additives In particular, at low concentration, when the particles are assumed to be deformable spheres equation (4 2) reduces to the following form as obtained by Taylor [see Rutgers, 1962]

$$\mu = \mu_o[1 + \lambda C_o]$$

where
$$\lambda = 2\ 5\ \frac{\mu_1 + \frac{2}{5}\mu_o}{\mu_1 + \mu_o} \tag{4 3}$$

It has also been pointed out that the concentration of the suspended particles changes when suspension flows Seshadri and Sutera [1968] Keeping this in view one may generalise equation (4 3) as follows

$$\mu = \mu_o[1 + \lambda C] \tag{4 4}$$

where C is the volume concentration governed by the equation of mass transfer and depends upon convection, chemical reaction etc present in the system This equation may also be assumed to be true when impurities are mixed in the lubricant during the operation of machines

In this chapter, a generalised theory of lubrication is formulated which includes the effects of additives/impurities present in the base lubricant during actual working conditions In particular, the effects of additives/impurities on the characteristics of slider and hydrostatic bearings are studied

4 1 BASIC EQUATIONS

In general, most of the lubricated systems can be considered to consist of two relatively moving surfaces with a thin film of lubricant between them The pressure generated in the film for a lubricant with variable viscosity is given by the generalised form of Reynolds equation, which is the outcome of equations of motion and the equation of continuity, under thin film approximation theory, Dowson [1962] This equation can be written as

$$\frac{\partial}{\partial x}\left(\Gamma \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(F \frac{\partial p}{\partial z}\right) = U \frac{\partial G}{\partial x} \qquad (4\ 5)$$

where

$$F = \int_0^h \frac{y^2}{\mu}\, dy \quad \Gamma \int_0^h \frac{y}{\mu}\, dy$$

$$\Gamma = \frac{\int_0^h \frac{y}{\mu}\, dy}{\int_0^h \frac{dy}{\mu}} \qquad (4\ 6)$$

and μ is given by equation (4 4) The volume concentration of the additive is governed by the equation of mass transfer and is written as follows by using thin film approximation theory, Bird et al [1960]

$$u \frac{\partial C}{\partial x} + w \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial y^2} + k' C \qquad (4\ 7)$$

where k' is the rate of loss or gain in C due to chemical reaction, absorption, adhesion, corrosion etc The velocities u

and w are given as follows Dowson [1962]

$$u = \frac{\partial p}{\partial x}\left[\int_0^y \frac{y}{\mu}\,dy \quad C\int_0^y \frac{dy}{\mu}\right] + U\left[1 - \frac{\int_0^y \frac{dy}{\mu}}{\int_0^h \frac{dy}{\mu}}\right]$$

$$w = \frac{\partial p}{\partial z}\left[\int_0^y \frac{y}{\mu}\,dy \quad G\int_0^y \frac{dy}{\mu}\right] \qquad (4\ 8)$$

Thus to study the effects of additives and impurities in lubricant, the system of equations (4 4) to (4 7) should be solved under suitable boundary conditions for p and C As equations (4 5) and (4 7) are inter-related and are highly non-linear in nature, only approximate solutions are possible In the following we solve these equations in simplified form to study the characteristics of slider and externally pressurised hydrostatic bearing

4 2 SLIDER BEARING

Let us consider the case of an infinite slider bearing as illustrated in figure No (4 1) It is assumed that there is no chemical reaction in the film but there is a possibility of greater mass transfer at the plate y = h by processes such as chemical reaction, adsorption or adhesion At the plate y = 0, the concentration of the additive/impurity is assumed to be prescribed They may be either due to additive present in the lubricant or due to wearing out of a certain solid lubricant coating on the surface To simplify the matter further, we assume that the diffusion due to

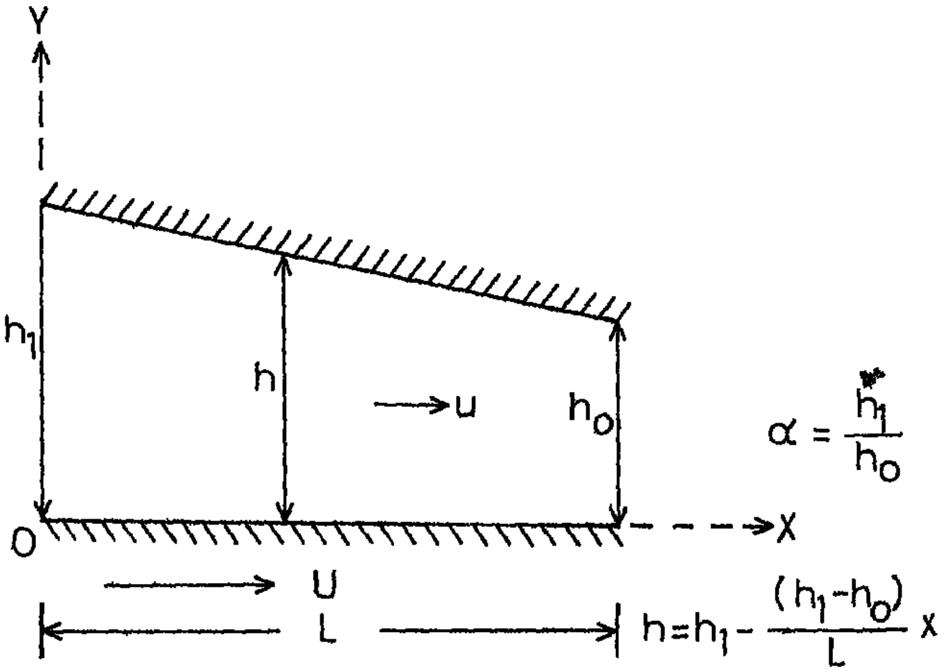


FIG 4.1 SLIDER BEARING WITH MASS TRANSFER AT y=h

convection and concentration changes with x are small In such a case, the equation (4 7) governing the concentration reduces to the following equation

$$\frac{d^2C}{dy^2} = 0 \tag{4 9}$$

with the following boundary conditions

$$C = C_o \quad \text{at} \quad y = 0$$

$$D\frac{dC}{dy} = k\,C \quad \text{at} \quad y = h = h_1 - \frac{(h_1 - h_o)}{L}\,x \tag{4 10}$$

Solving equation (4 9) and using conditions (4 10) we get a linear concentration profile as follows

$$C = C_o\left[1 + \frac{ky}{D-kh}\right] \tag{4 11}$$

If there is no mass transfer at $y = h$, i e when $k = 0$, then the system behaves as if the bearing is lubricated with an additive of uniform volume concentration C_o

After C is determined we can write the modified viscosity of the base lubricant by using equation (4 4) as follows

$$\mu = \mu_o\left[1 + \lambda C_o\left(1 + \frac{ky}{D-kh}\right)\right] \tag{4 12}$$

With this expression for μ, the Reynolds equation (4 5) in this case is written as

$$\frac{dp}{dx} = -\frac{Q}{F} + U\frac{G}{F} \tag{4 13}$$

where the flow flux $Q = \int_0^h u \; dy$ = a constant

Solving equation (4 13) and using the usual boundary condition $p = 0$ at $x = 0$ and $p = 0$ at $x = L$, we get the expressions for pressure and flow flux as follows :

$$p = U\left[\int_0^x \frac{G}{F}dx - \frac{\int_0^L \frac{G}{F}dx}{\int_0^L \frac{1}{F}dx}\int_0^x \frac{1}{F}dx\right] \tag{4 14}$$

$$Q = U\frac{\int_0^L \frac{G}{F}dx}{\int_0^L \frac{1}{F}dx} \tag{4 15}$$

Also the expressions for load capacity and frictional force at the moving plate are given respectively as follows by using equations (4 14) and (4 8)

$$W = U\left[\frac{Q}{U}\int_0^L \frac{x}{F}dx - \int_0^L \frac{xG}{F}dx\right] \tag{4 16}$$

$$T = U\left[\frac{Q}{U}\int_0^L \frac{G}{F}dx - \int_0^L \frac{G^2}{F}dx - \int_0^L \frac{dx}{\int_0^h \frac{dy}{\mu}}\right] \tag{4 17}$$

Since the concentration of additives or impurities are fairly small, the functions F and G in above integrals may be approximated as follows

$$F = \frac{h^3}{12\mu_o} [1 - \lambda C_o \ (1 + \frac{1}{2} \ \frac{kh}{D-kh} \ )]$$

$$G = \frac{h}{2} [1 - \frac{\lambda C_o}{6} \ \frac{kh}{D-kh}] \tag{4 18}$$

With these expressions for F and G, the expressions for flow rate, load capacity and frictional force are given as follows

$$\bar{Q} = \frac{\alpha}{\alpha+1} + \lambda C_o \ Q_C \tag{4 19}$$

$$\bar{W} = \frac{6 \ \ell n \ \alpha}{(\alpha-1)^2} - \frac{12}{\alpha^2-1} + \lambda C_o \ W_C \tag{4 20}$$

$$\bar{T} = \frac{6}{\alpha+1} - \frac{4 \ \ell n \ \alpha}{\alpha-1} + \lambda C_o \ T_C \tag{4 21}$$

where

$$\alpha = \frac{h_1}{h_o}, K = \frac{kh_o}{D}, \ \bar{Q} = \frac{Q}{Uh_o}, \ \bar{W} = \frac{Wh_o^2}{\mu_o UL^2}, \ \bar{T} = \frac{Th_o}{\mu_o LU}$$

$$Q_C = \frac{\alpha}{\alpha+1} \ \{\frac{\alpha KA}{\alpha-1} (\frac{1}{3} \quad \frac{\alpha K}{\alpha+1}) - \frac{\alpha K}{\alpha+1}\} \tag{4 22}$$

$$W_C = \frac{6}{(\alpha-1)^2} \ [\frac{1}{\alpha+1} \ \{(\alpha-1)^2(1+Q_C) + \alpha P\} - C] \tag{4 23}$$

$$T_C = \frac{6}{\alpha+1} \ (1+Q_C) + A \ \{\frac{2\alpha K}{\alpha^2-1} - \frac{4}{\alpha-1}\} \tag{4 24}$$

$$A = \ln \frac{\alpha(1\ \ )}{1\ \alpha\lambda}$$

$$B = I(\alpha-1) + A\gamma(\alpha\lambda-1)$$

$$C = \alpha-1\quad \frac{2}{3}\ln\alpha - \frac{A}{3}(1 - \alpha\gamma)$$

When $K \to 0$, from equations (4 19), (4 20) and (4 21) the corresponding expressions for flow flux, load capacity and frictional force respectively are obtained as follows

$$\bar{Q} = \frac{\alpha}{\alpha+1} \tag{4 25}$$

$$\bar{W} = \{\frac{6\ \ln\alpha}{(\alpha\ 1)^2} - \frac{12}{\alpha^2\ 1}\}\ (1 + \lambda C_o) \tag{4 26}$$

$$\bar{T} = \{\frac{6}{\alpha+1} - \frac{4\ \ln\alpha}{\alpha\ 1}\}\ (1 + \lambda C_o) \tag{4 27}$$

The variations of Q_C, $\bar{Q}$, W_C, $\bar{W}$, $(-T_C)$ and $(-\bar{T})$ are shown in figures (4 2) to (4 7) respectively It is noted from figures (4 2), (4 4) and (4 6) that Q_C (the contribution to the flux due to mass transfer), decreases while W_C (the contribution to the load due to mass transfer) and $(-T_C)$ (the contribution to frictional force at the moving plate due to mass transfer) increase as K, the mass transfer parameter, increases From figures (4 3), (4 5) and (4 7) it is obvious that $\bar{Q}$ decreases and $\bar{W}$, $(-\bar{T})$ increase with the increase in K for fixed λC_o

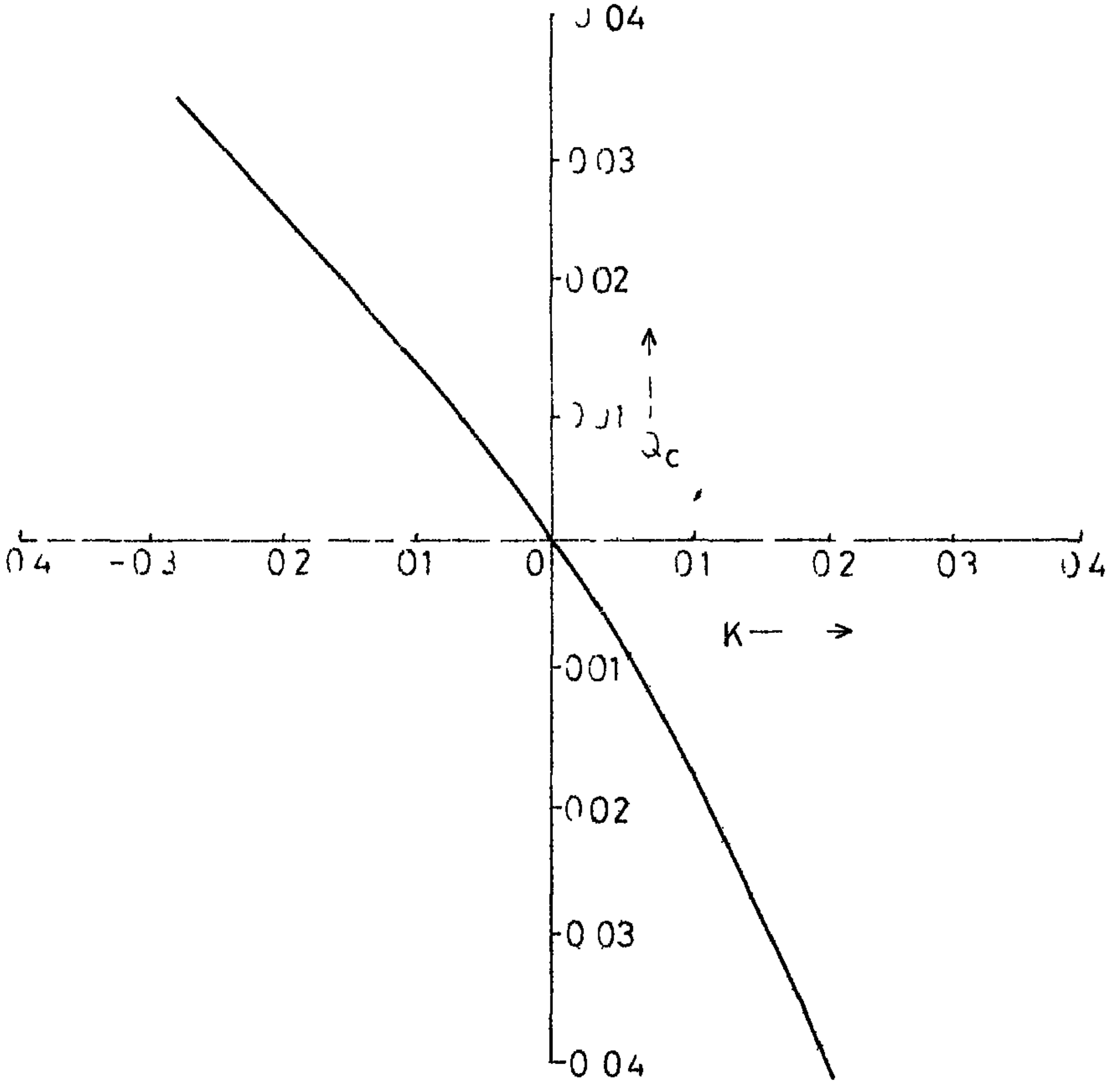


FIG 4 2 VARIATION OF Q_c WITH K FOR $\alpha = 2$

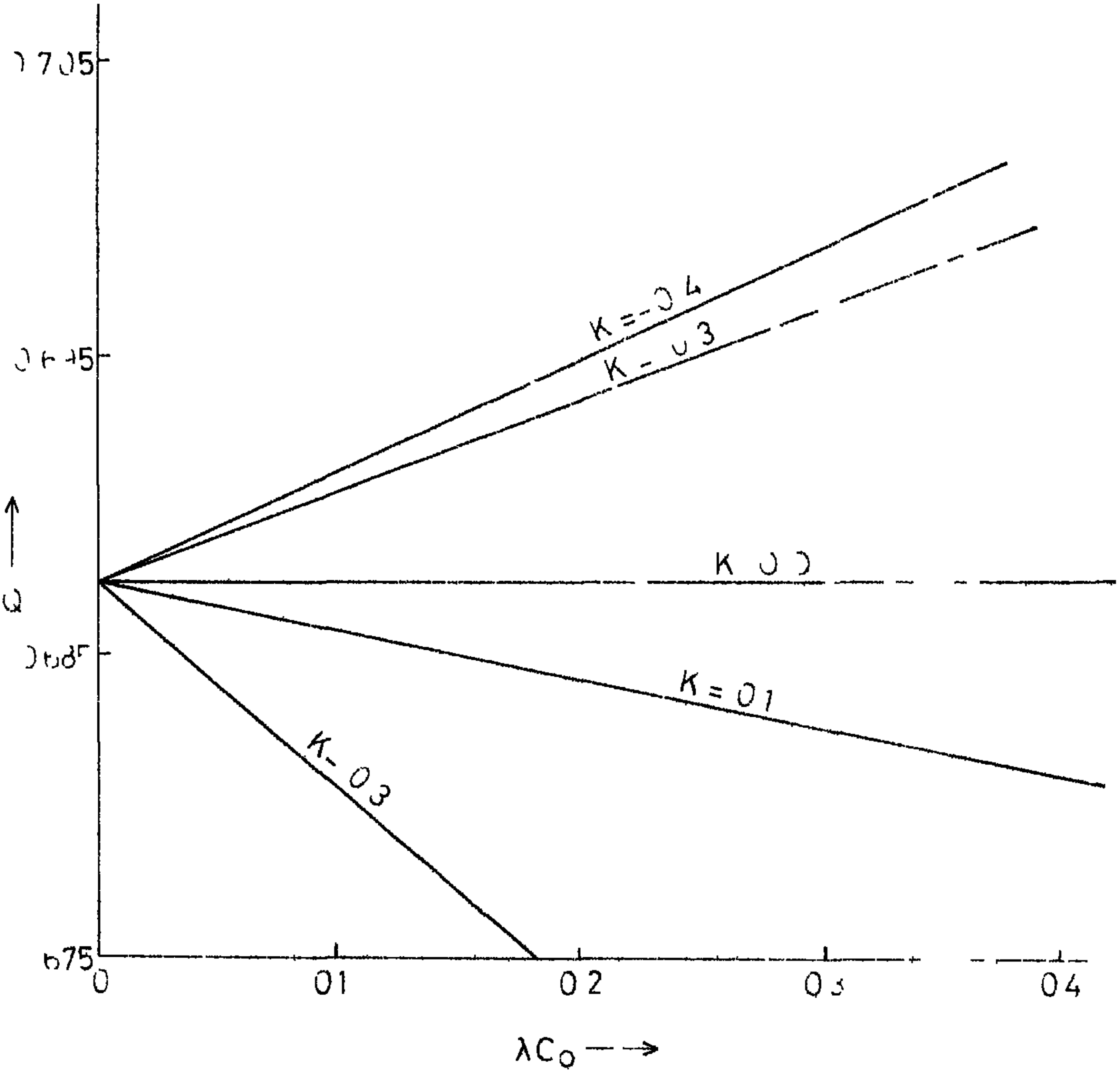


FIG 4 3 VARIATION OF $\bar{Q}$ WITH λC_o FOR VARIOUS VALUES OF K AND α = 2 2

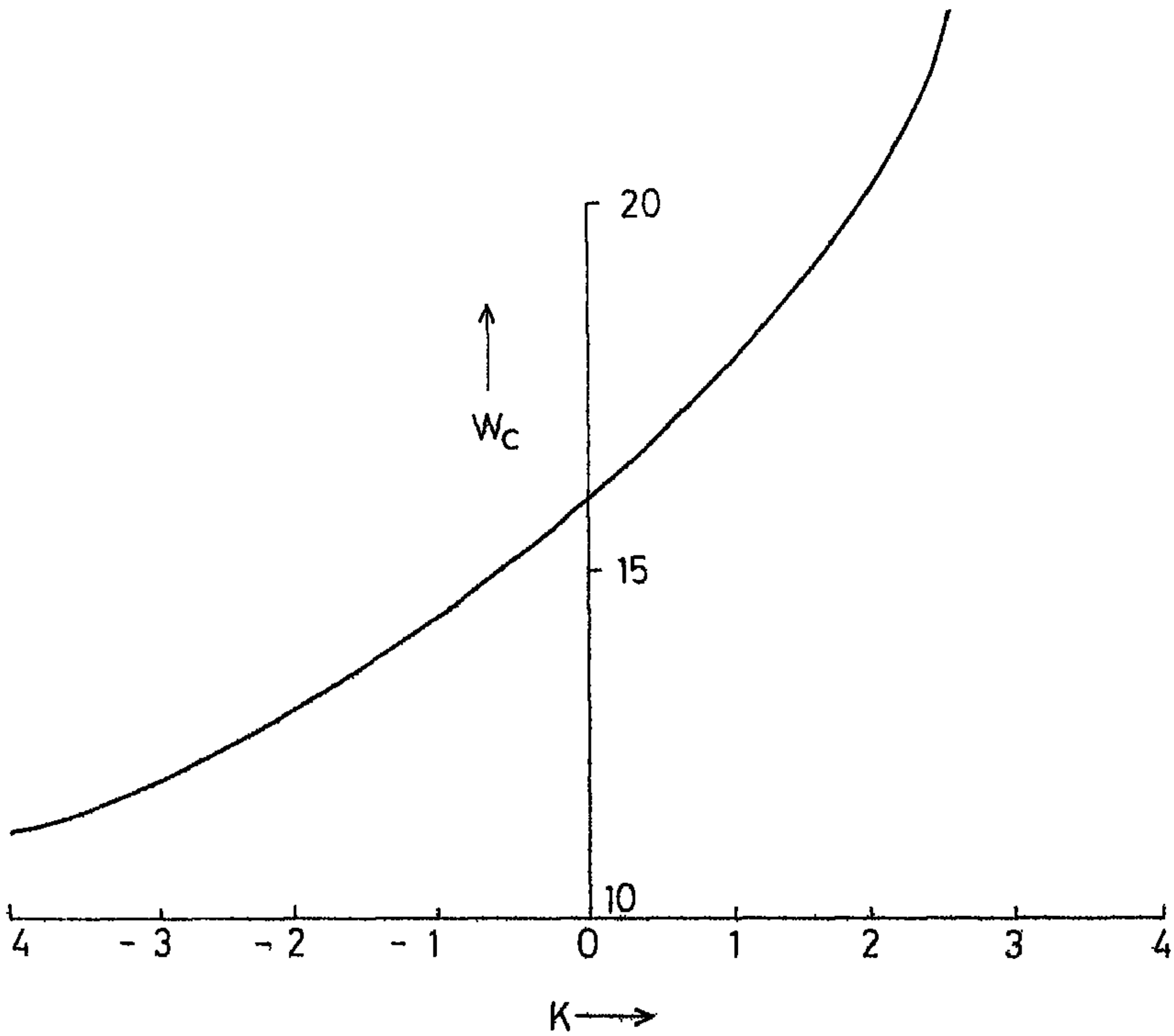


FIG 4 4 VARIATION OF W_c WITH K FOR α = 2 2

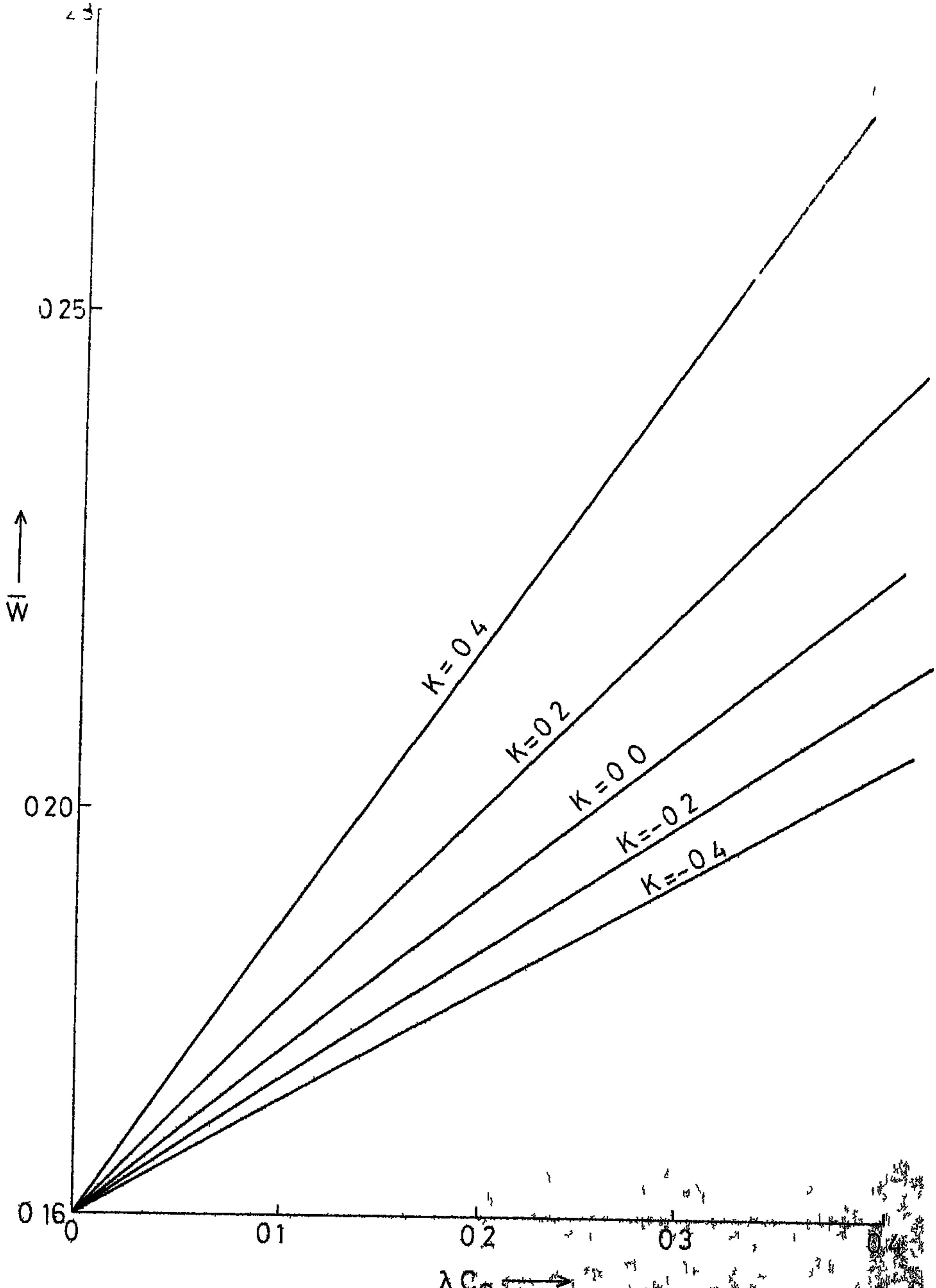


FIG 4 5 VARIATION OF $\overline{W}$ WITH λC_0 FOR VARIOUS K AND $\alpha = 2\ 2$

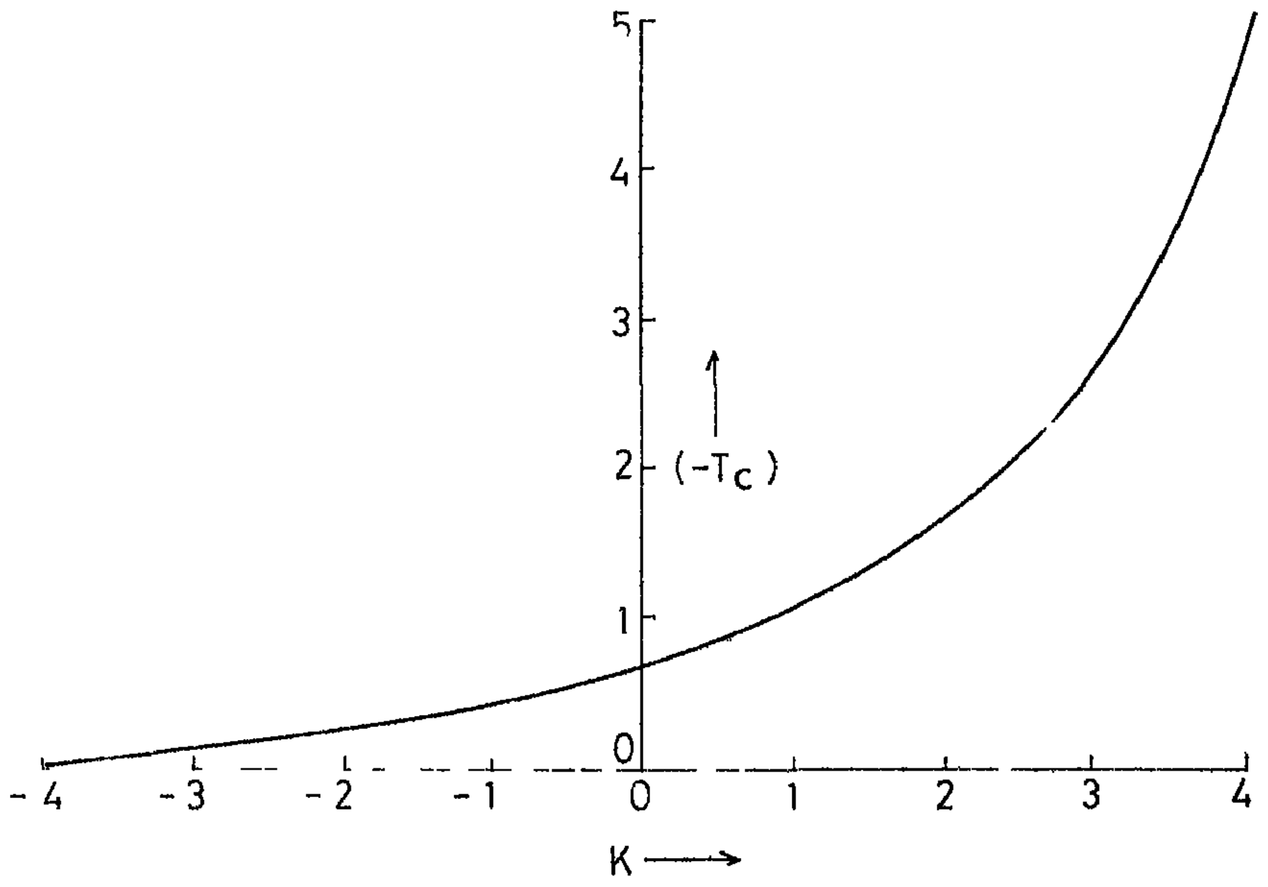


FIG 4 6 VARIATION OF $(-T_c)$ WITH K FOR $\alpha = 2.2$

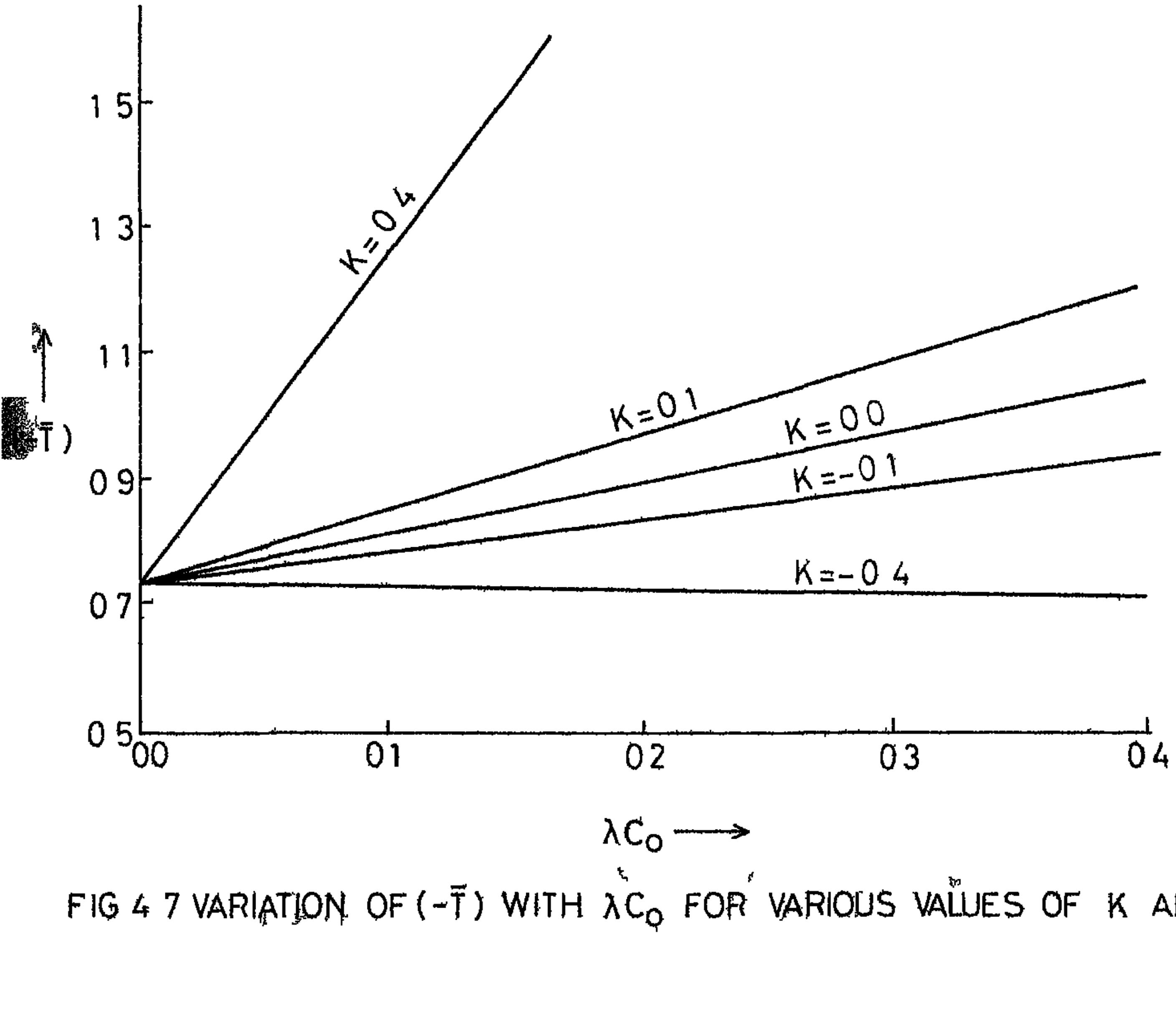


FIG 4 7 VARIATION OF $(-\bar{T})$ WITH λC_0 FOR VARIOUS VALUES OF K AND $\alpha=22$

The behaviour of $\bar{Q}$, $\bar{W}$ and $(-\bar{T})$ are different with λC_0 for each K For example $\bar{Q}$ is a constant when $K = 0$ and decreases for positive K and increases for negative K with the increase in λC_0 [see figure no (4 3)] But $\bar{W}$ and $(-\bar{T})$ increase with λC_0 for all values of K as shown in figures(4 5) and (4 7)

4 3 HYDROSTATIC BEARING

Consider the case of a parallel surface hydrostatic bearing as explained in figure no (4 8) It is assumed that the mass is being transferred at the plates and also there is chemical reaction in the film itself, may be due to presence of some detergent In such a case, the equation determining the concentration is written as follows [see equation (4 7)]

$$u \frac{\partial C}{\partial r} = D \frac{\partial^2 C}{\partial z^2} + \quad {}'C \qquad (4\ 28)$$

where k' is the rate of first-order chemical reaction and is negative if there is a loss in C, otherwise positive

To obtain simple solution of equation (4 28) we assume that the concentration gradient in the axial direction is large but C does not vary very much across the film This is possible because concentration gradient in axial direction results from small concentration changes over extremely small axial distances, Sneck [1969] With this, taking average of equation (4 28) we have,

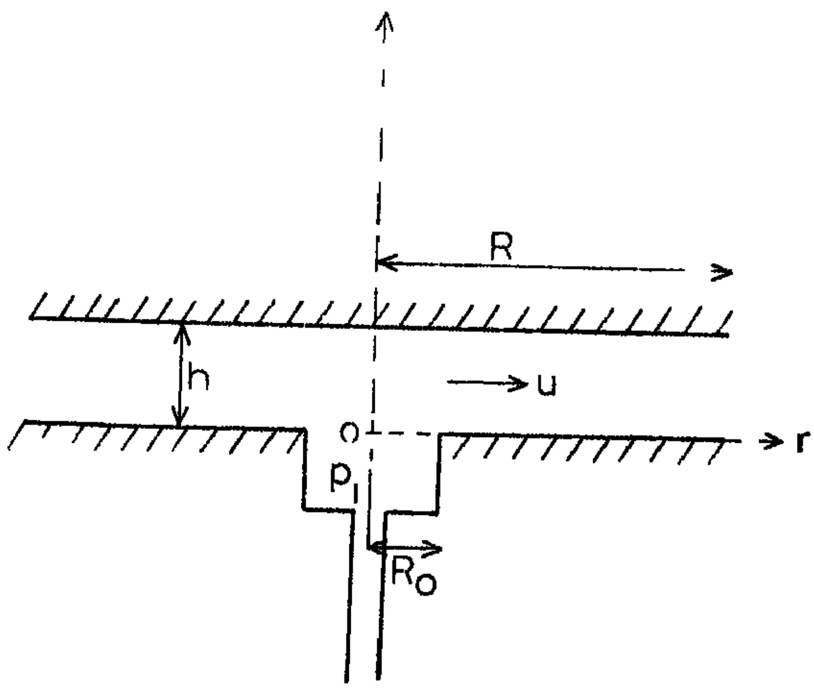


FIG 4 8 EXTERNALLY PRESSURISED HYDROSTATIC BEARING WITH MASS TRANSFER

$$Q \frac{dC}{dr} = 2\pi r C \; (k'h + k_1) \tag{4 29}$$

Here Q is the flow flux of the lubricant and is given by

$$Q = \int_0^h 2\pi u r \; dz = \text{constant} \tag{4 30}$$

and k_1 is given by

$$(D \frac{\partial C}{\partial z})_h - (D \frac{\partial C}{\partial z})_o = k_1 C \tag{4 31}$$

This equation represents the mass balance at surfaces Thus, the equation (4 29) may be interpreted as the balance of mass transfer in a thin circular film due to convection and due to chemical reaction in and around it

Solving equation (4 29) and using the boundary condition $C = C_o$, at $r = R_o$, we get

$$C = C_o e^{\frac{\pi}{Q}(k'h + k_1)(r^2 - R_o^2)} \tag{4 32}$$

Then the average viscosity of the suspension is written as follows :

$$\mu = \mu_o \left[1 + \lambda C_o \; \text{Exp} \; (K' + K_1) \left(\frac{r^2}{R^2} - \bar{R}^2 \right)\right] \tag{4 33}$$

where

$$K' = \frac{k'h\pi R^2}{Q} , \quad \bar{R} = \frac{R_o}{R}$$

$$K_1 = \frac{k_1 \pi R^2}{Q}$$

Now the equation determining the pressure in this case is given by

$$\frac{dp}{dr} = - \frac{6\mu Q}{\pi r h^3} , \tag{4 34}$$

where μ is given by equation (4 33) Integrating equation (4 34) and using the boundary condition $p = p_i$ at $r = R_o$ and $p = 0$ at $r = R$, we get

$$p = - \frac{6Q}{\pi h^3} \int_R^r \frac{\mu}{r} dr \tag{4 35}$$

$$p_i = \frac{6Q}{\pi h^3} \int_{R_o}^R \frac{\mu}{r} dr \tag{4 36}$$

The load capacity of the bearing for a constant flux is given by

$$W = \frac{6Q}{h^3} \int_{R_o}^R \mu r \, dr \tag{4 37}$$

which on using equation (4 33) gives,

$$\bar{W} = 3 \left(1 - \bar{R}^2\right) + \lambda C_o W_C' , \tag{4 38}$$

where $\bar{W} = \dfrac{Wh^2}{\mu_o Q R^2}$

and

$$W_C' = \frac{3}{(K'+K)} \; [\mathrm{Exp}\;\{(K'+K_1)(1-\bar{R}^2)\} - 1] \tag{4 39}$$

When $K' + K_1 \to 0$, we get

$$\bar{W} = 3\;(1-\bar{R}^2)\;(1+\lambda C_o) \tag{4 40}$$

The variations of W_C' (the contribution to the load capacity due to mass transfer) and $\bar{W}$ are shown in figures (4 9) and (4 10) respectively It is obvious from these figures that both W_C' and $\bar{W}$ increase as $K' + K_1$ increases The load capacity also increases with the increase in λC_o for all values of $K' + K_1$ as shown in figure no (4 10)

Similarly the expression determining the torque M is given by

$$M = \frac{2\pi\Omega}{h} \int_{R_o}^{R} \mu r^3 \, dr$$

which on using equation (4 33) again, gives

$$\bar{M} = \frac{1}{2}\,(1-\bar{R}^4) + \lambda C_o M_C \tag{4 41}$$

where

$$\bar{M} = \frac{Mh}{\pi\Omega\mu_o R^4}$$

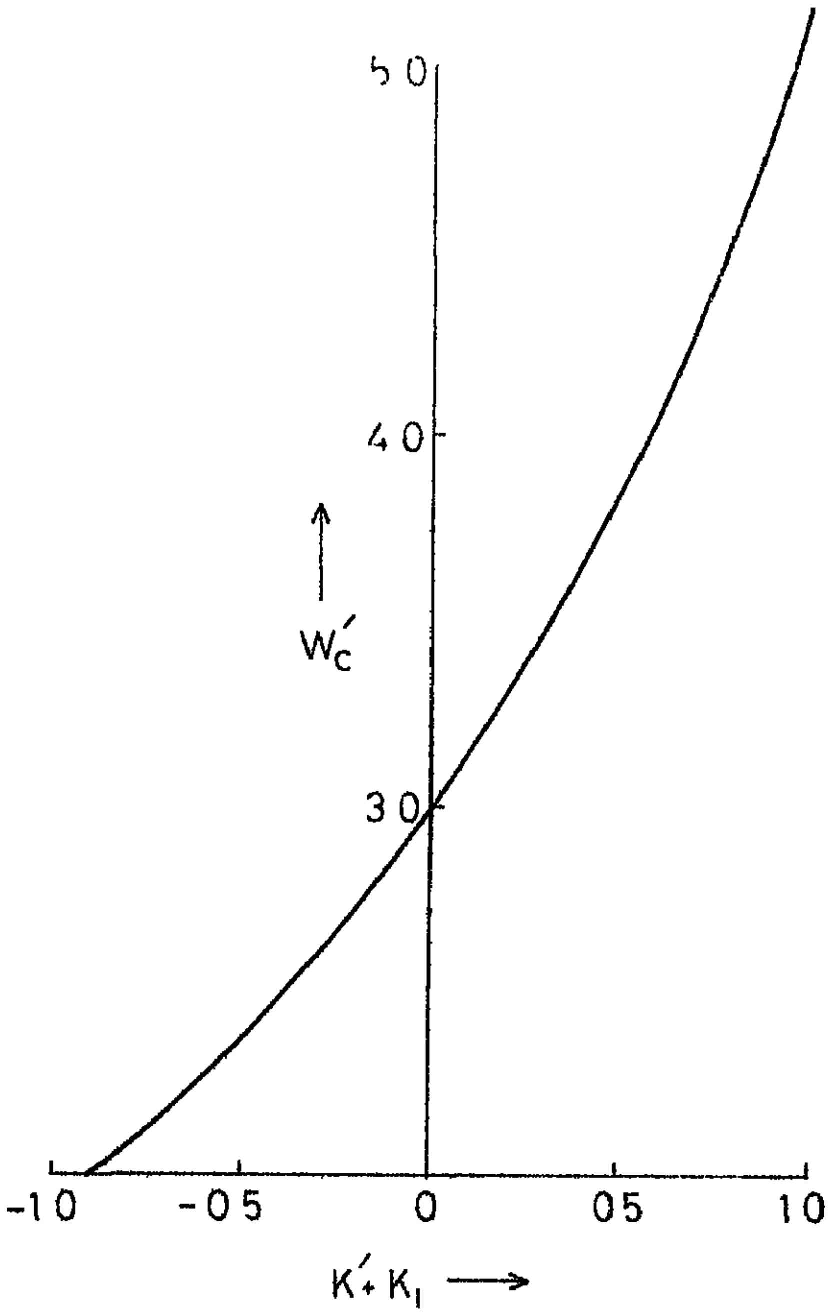


FIG 4 9 VARIATION OF W_c' WITH $K' + K_1$ FOR $\bar{R} = 0\ 05$

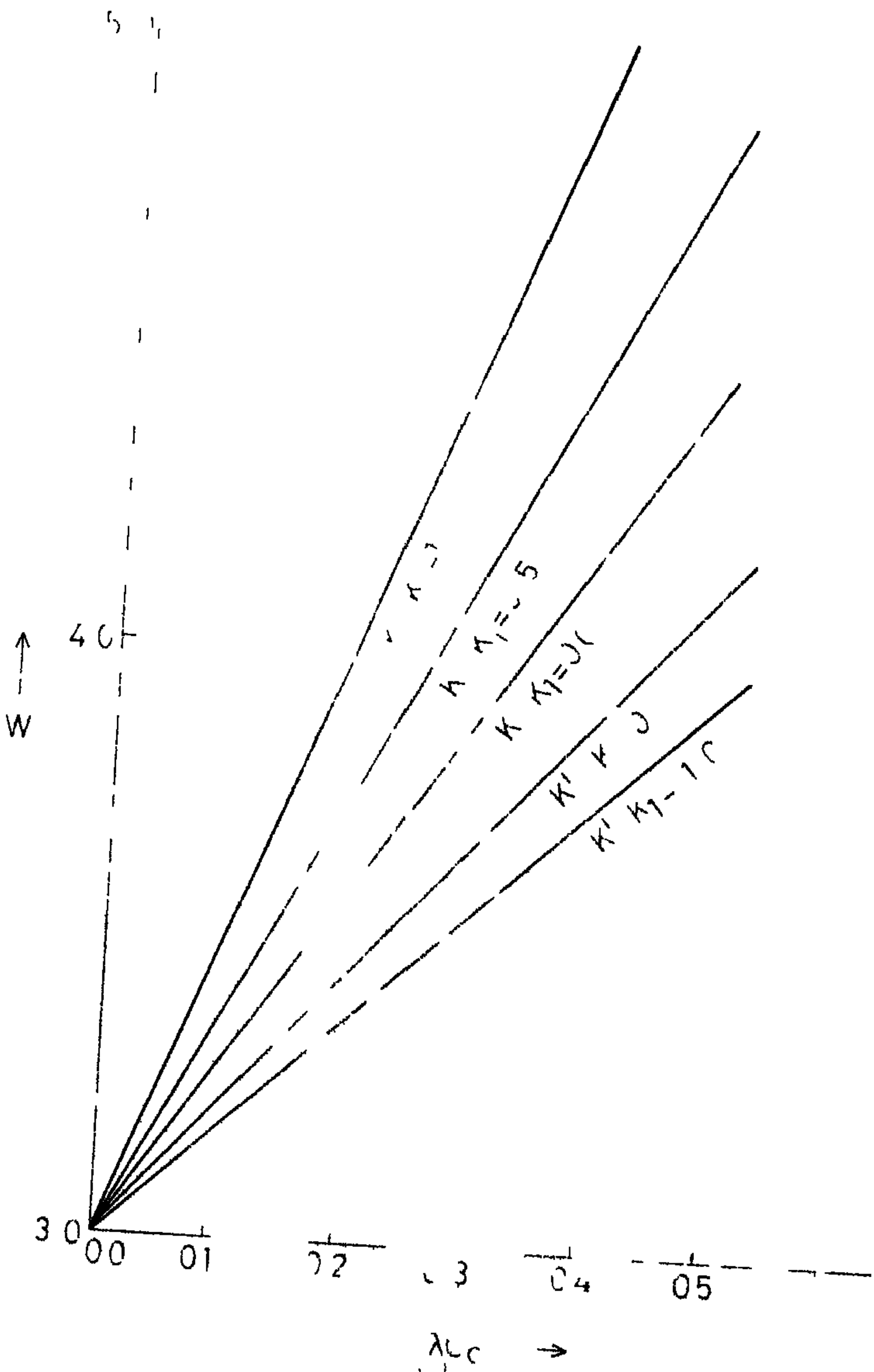


FIG 4 10 VARIATION OF $\bar{W}$ WITH λC_0 FOR VARIOUS VALUES OF $K' + K_1$ AND $\bar{R} = 0.05$

and

$$M_0 = \frac{1}{(K' + K_1)} [(1 - \frac{1}{K'+K_1}) \text{Exp} \{(K'+K_1)(1-\bar{R}^2)\} + \frac{1}{(K'+K_1)} - \bar{R}^2] \tag{4 42}$$

When $K' + K_1 \to 0$, we have

$$\bar{M} = \frac{1}{2}(1 - \bar{R}^4)(1 + \lambda C_0) \tag{4 43}$$

The behaviour of M_0 and $\bar{M}$ are shown in figures (4 11) and (4 12) respectively It is seen that both M_0 and $\bar{M}$ increase with $K' + K_1$ but $\bar{M}$ also increases with λC_0 for all values of $K' + K_1$

4 4 RESULTS AND DISCUSSION

It is seen that the presence of additives and impurities in the base lubricant is an important factor in determining the characteristics of the bearing The presence of mass transfer process in the lubricated system can also modify the behaviour considerably In the case of slider bearing it is shown that the load capacity and the magnitude of frictional force at the moving surface increase with λC_0, and K, the mass transfer parameter The flow flux, however, decreases with K for fixed λC_0 Similar results for the load capacity and frictional torque in the case of hydrostatic bearing are also proved.

Finally it may be remarked that if the effect of mass transfer (K or $K' + K_1$) on the surfaces or in the film is such that it increases

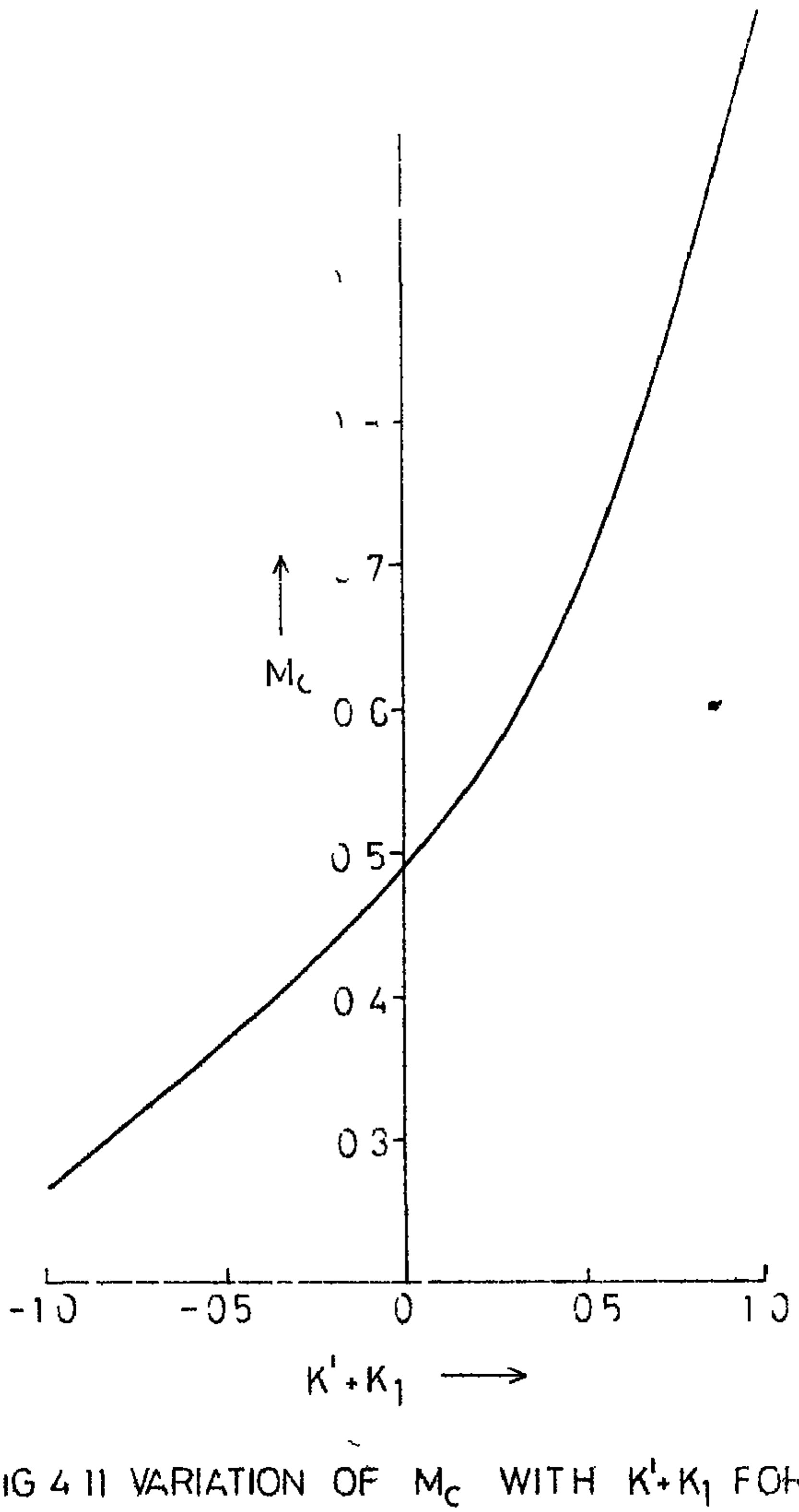


FIG 4 11 VARIATION OF M_c WITH $K'+K_1$ FOR R - 5

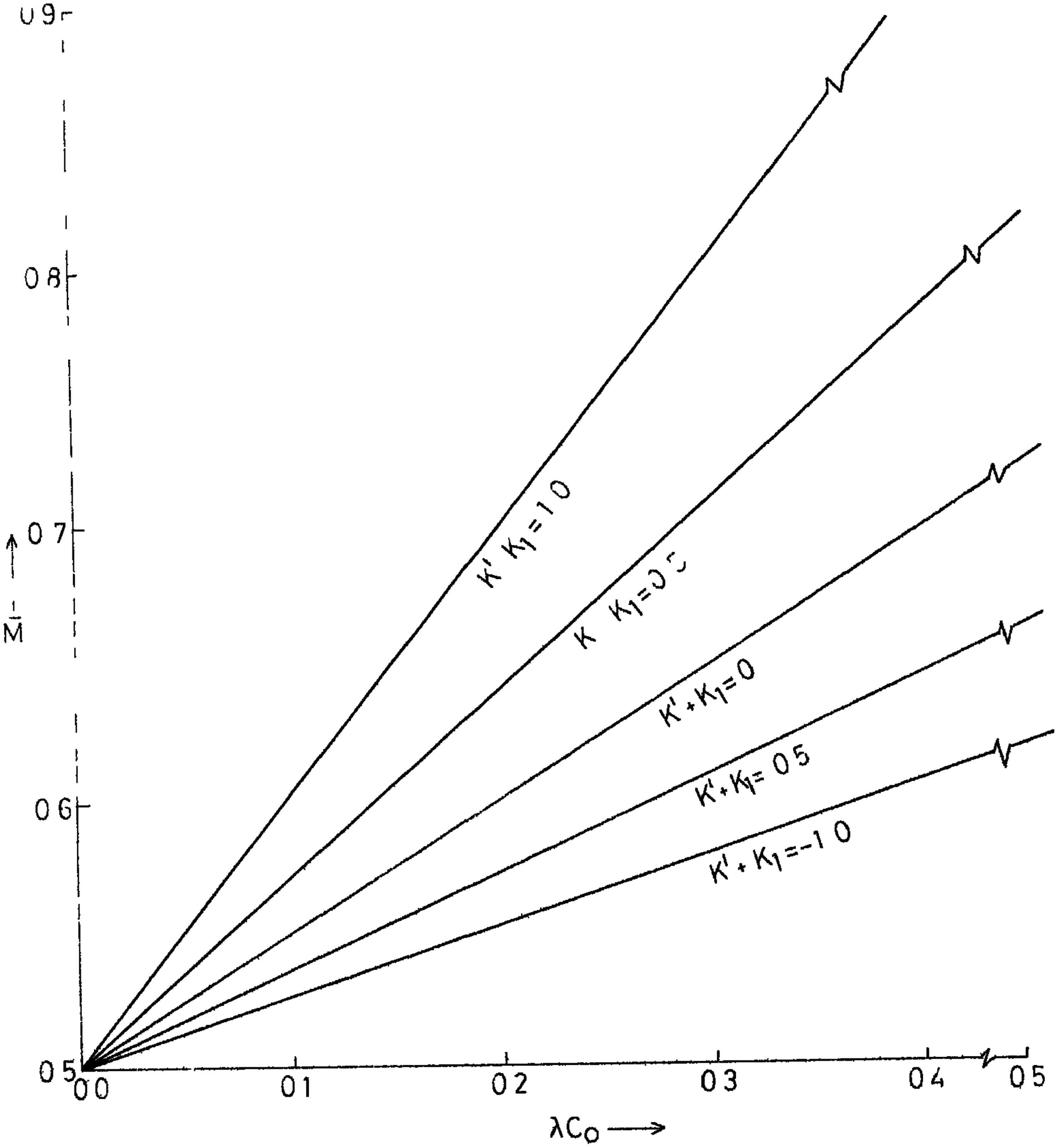


FIG 4 12 VARIATION OF $\bar{M}$ WITH λC_0 FOR VARIOUS VALUES OF $K' + K_1$ AND $\bar{R} = 0\ 05$

the concentration of additives or impurities in the base lubricant, then the load capacity and the frictional force would increase in the above mentioned system

However, if the effects of K or $K' + K_1$ are such that it decreases the overall concentration of the additive in the base lubricant, then the load capacity and frictional force would decrease in comparison to the case of original concentration Such a situation arises when the additives get lost on the surfaces or in the film because of chemical reaction due to the presence of detergent etc

CHAPTER - V

CHARACTERISTICS OF NON-NEWTONIAN POWER LAW LUBRICANTS IN AN EXTERNALLY PRESSURISED CONICAL STEP BEARING AND IN HYDROSTATIC STEP SEAL

As pointed out in Chapter I, the lubricants which do not follow Newtonian hypothesis i e a linear relationship between shearing stress and rate of shear are called non-Newtonian Such non-Newtonian lubricants for which shear stress varies as some power of shear-rate are called Ostwald-DeWaale or Power law models The deviation of this power index n(say) from unity determines the change in Newtonian behaviour of the lubricant For $n = 1$, the fluid behaves as Newtonian, for $n > 1$ it behaves as a dilatant fluid and for $n < 1$ it characterises the pseudoplastic behaviour

The characteristics of various bearings with power law fluids as lubricants have been studied and the deviations from Newtonian behaviour have been pointed out , Ng and Saibel [1962] , Shukla [1963a, 1964b, 1964c], Hsu and Saibel [1965] , Tanner [1965] , Shukla and Prakash [1969] However, very little interest has been shown to study the characteristics of such fluids in hydrostatic or hydrodynamic seals, Tanner [1960] , Cheng, Chow and Wilcock [1968]

In this chapter, the uses of non-Newtonian power law fluids as lubricants are investigated in the following systems The effects of stepped film thickness on the various characteristics of these systems are pointed out

I Externally pressurised conical step bearing

II Hydrostatic step seal

5 1 BASIC EQUATION FOR THE FLOW OF POWER LAW LUBRICANTS IN A THIN FILM

Consider the flow of a power law fluid in an infinitely long thin clearance eith r due to external pre surisation or due to normal motion of the plates In such a case the velocity profile would be as shown in figure no (5 1)

The stress-strain relation characterising the power law fluid is given by

$$\tau_{xy} = m \left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y} \tag{5 1}$$

where m,n are the consistency and flow behaviour indices of the fluid respectively The equation governing the velocity of the fluid under the assumptions of lubrication theory is given by

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{dp}{dx} \tag{5 2}$$

From equations (5 1) and (5 2), we get

$$m \frac{\partial}{\partial y} \left\{\left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}\right\} = \frac{dp}{dx} \tag{5 3}$$

Let $y = \nu h$ be the height where the velocity u of the fluid is maximum Then

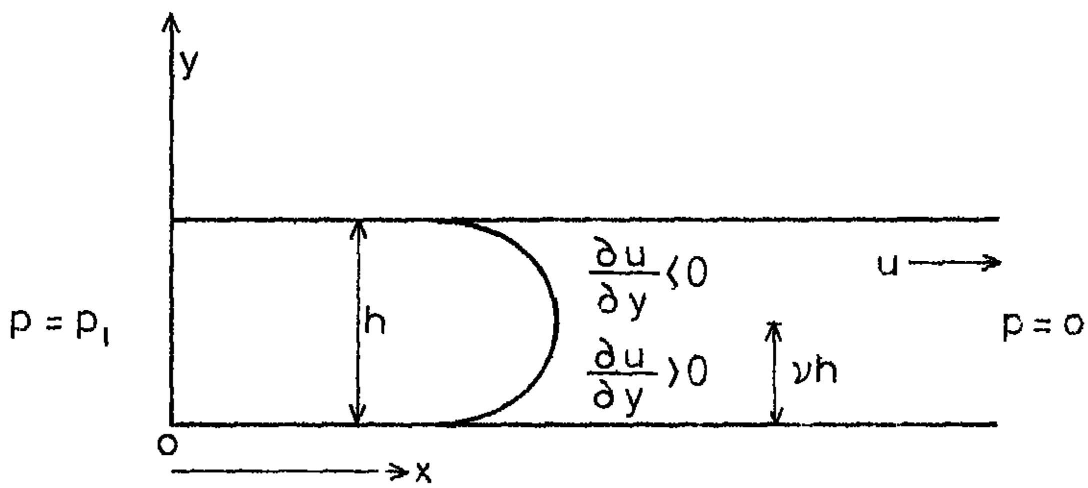


FIG 5 1 FLOW OF A POWER LAW FLUID IN A THIN CLEARANCE

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = \nu h \tag{5 4}$$

Now in the region $\nu h \leq y \leq h$, clearly $\frac{\partial u}{\partial y} \leq 0$, and the equation governing the velocity of the fluid [from equations (5 1), (5 2) and (5 3)] is given by

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n = -\frac{1}{m}\frac{dp}{dx}, \quad \nu h \leq y \leq h \tag{5 5}$$

Integrating equation (5 5) and applying condition (5 4) we have

$$\frac{\partial u}{\partial y} = \left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n} (y - \nu h)^{1/n} \tag{5 6}$$

Integrating equation (5 6) again and using the no slip condition $u = 0$ at $y = h$, we get the final expression for the velocity of the lubricant as follows

$$u = \left(\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} \frac{(h-\nu h)^{\frac{1}{n}+1} - (y-\nu h)^{\frac{1}{n}+1}}{1+\frac{1}{n}} \quad \nu h \leq y \leq h \tag{5 7}$$

Similarly, in the region $0 \leq y \leq \nu h$, we have $\frac{\partial u}{\partial y} \geq 0$, and the equation governing the velocity is as follows:

$$\frac{\partial}{\partial y}\left\{\left(\frac{\partial u}{\partial y}\right)^n\right\} = \frac{1}{m}\frac{dp}{dx}, \quad 0 \leq y \leq \nu h \tag{5 8}$$

Integrating equation (5 8) with condition (5 4) and $u = 0$ at $y = 0$, we have

$$\frac{\partial u}{\partial y} = \left(\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} (y - \nu h)^{\frac{1}{n}} \tag{5.9}$$

$$u = -\left(\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} \frac{(\nu h - y)^{1+\frac{1}{n}} - (\nu h)^{1+\frac{1}{n}}}{1+\frac{1}{n}} \qquad 0 \leq y \leq \nu h \tag{5.10}$$

Now at $y = \nu h$ the velocities are continuous, so from equations (5.7) and (5.10) we get

$$\left(-\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} \frac{h^{1+\frac{1}{n}}}{1+\frac{1}{n}} (1-\nu)^{1+\frac{1}{n}} = \left(-\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} \frac{h^{1+\frac{1}{n}}}{1+\frac{1}{n}} \nu^{1+\frac{1}{n}} \tag{5.11}$$

where each side of the above equation gives the maximum velocity of the fluid

Equation (5.11) implies that $\nu = \frac{1}{2}$ and the flow is symmetrical about the middle point of the film-thickness

The volume flux Q of the fluid is defined by

$$Q = b\left\{\int_0^{h/2} u\,dy + \int_{h/2}^{h} u\,dy\right\} \tag{5.12}$$

which on using equations (5.7) and (5.10) gives

$$Q = \frac{2nb}{2n+1}\left(-\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} (h/2)^{2+\frac{1}{n}} \tag{5.13}$$

When the flow is due to squeezing, on integrating the equation of continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ and using the boundary conditions $v = -V$ at $y = h$ and $v = 0$ at $y = 0$, we have

$$V = \frac{\partial}{\partial x}\int_0^h u\,dy \qquad (5\ 14)$$

From equations (5 12) and (5 14) we have the differential equation for determining the flux as follows

$$\frac{\partial Q}{\partial x} = b\ V \qquad (5\ 15)$$

If there is no squeezing i e $V = 0$ and the flow is due to external pressurisation, Q is a constant

5 2 CASE I EXTERNALLY PRESSURISED CONICAL STEP BEARING

Consider the case of externally pressurised conical bearing with stepped film thickness using a power law fluid as lubricant The physical situation is shown in figure no (5 2)

In this case the equation determining the pressure can be written [from equation (5 13) and putting $b = 2\pi\ x \sin\alpha$, and $Q_1 = Q_2 = Q$ (a constant)] as follows

$$\frac{dp_j}{dx} = -m\left\{\frac{(2n+1)Q}{4n\pi\ \sin\alpha}\right\}^n\left(\frac{2}{h_j}\right)^{2n+1}\frac{1}{x^n}\ ,\ j = 1,2\ , \qquad (5\ 16)$$

where Q is the flow flux and p_1 and p_2 are the pressures in the regions $\{h = h_1, k_0L \le x \le kL\}$ and $\{h = h_2,\ kL \le x \le L\}$ respectively

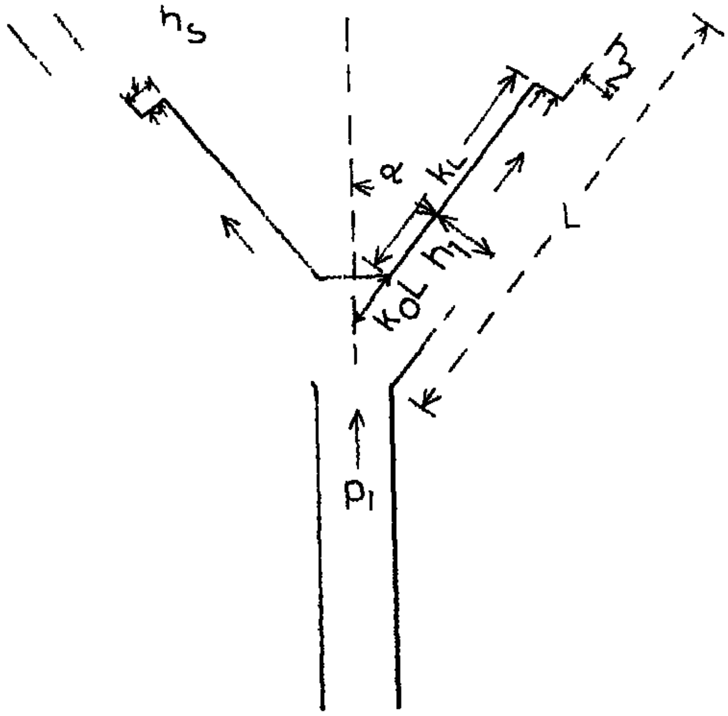


FIG 5.2 EXTERNALLY PRESSURISED CONICAL STEP BEARING

The boundary conditions are

$$
\begin{aligned}
p_1 &= p_i && \text{at} \quad x = k_o L \\
p_1 &= p_2 = p_s && \text{at} \quad x = kL \\
p_2 &= 0 && \text{at} \quad x = L
\end{aligned}
\tag{5 17}
$$

where p_s is the pressure at the step

Integrating equations (5 16) and using the boundary conditions (5 17) we have

$$p_1 = \frac{p_i}{g} \left[\left\{ k^{1-n} - \left(\frac{x}{L} \right)^{1-n} \right\} + H^{2n+1} (1-k^{1-n}) \right] \tag{5 18}$$

and $$p_2 = \frac{p_i}{g} H^{2n+1} \left[1 - \left(\frac{x}{L} \right)^{1-n} \right] \tag{5 19}$$

where

$$g = (k^{1-n} - k_o^{1-n}) + H^{2n+1}(1 - k^{1-n}),$$

$$H = \frac{h_1}{h_2} = \frac{h_2 + h_s}{h_2} = 1 + \frac{h_s}{h_2} \tag{5 20}$$

Further, the pressure at the step and the flow flux are given by

$$p_s = \frac{p_i}{g} H^{2n+1} (1 - k^{1-n}) \tag{5 21}$$

$$Q = \left[\frac{p_i (1-n)}{mg \, L^{1-n}} \right]^{\frac{1}{n}} \left(\frac{h_1}{2} \right)^{2 + \frac{1}{n}} \frac{4n\pi \sin \alpha}{2n+1} \tag{5 22}$$

When $h_s = 0$, i e $H = 1$ the corresponding expression for the flow flux Q_o is given by

$$Q_o = \left[\frac{p_1(1-n)}{mL^{1-n}(1-k_o^{1-n})}\right]^{\frac{1}{n}} \left(\frac{h_2}{2}\right)^{2+\frac{1}{n}} \frac{4n\pi \sin\alpha}{2n+1} \qquad (5\ 23)$$

The ratio of $\frac{Q_o}{Q}$ with respect to H is shown in figure no. (5 3).
It can be seen from this graph that $\frac{Q_o}{Q}$ decreases as H or h_s increases and $Q \geq Q_o$ for all n Since Q_o is not a function of h_s, it implies that Q increases as H or h_s increases

The load capacity of the bearing is given by

$$W_o = \pi p_1 k_o^2 L^2 \sin^2\alpha + \int_{k_oL}^{kL} 2\pi x \ \sin\alpha \ \ p_1 \sin\alpha \ \ dx$$

$$+ \ 2\pi kL \sin\alpha \, p_s \, h_s \cos\alpha + \int_{kL}^{L} 2\pi x \sin\alpha \ \ p_2 \sin\alpha \, dx \qquad (5\ 24)$$

Substituting the values of p_1, p_2 and p_s from equations (5 18), (5 19) and (5 21) respectively in equation (5 24), we have the final expression for the load capacity as follows:

$$\bar{W}_o = k_o^2 \sin^2\alpha + \frac{2}{g}\bar{h}_2(H-1) \ H^{2n+1} k(1-k^{1-n}) \sin\alpha \cos\alpha$$

$$+ \frac{2F_o}{g} \sin^2\alpha \quad , \qquad (5\ 25)$$

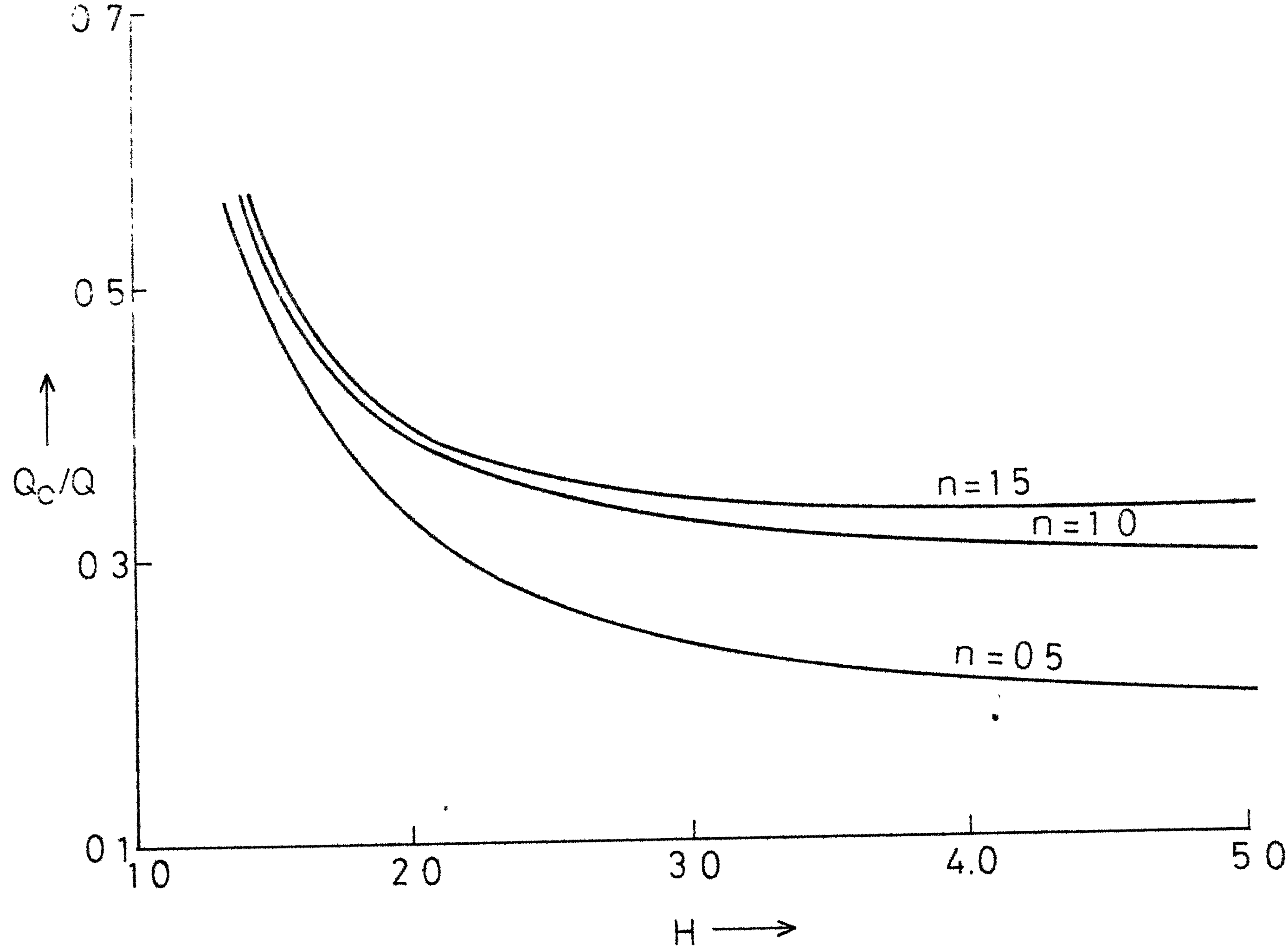


FIG 5 3. VARIATION OF Q_o/Q IN CASE OF CONICAL BEARING WITH $k = 0\ 5$, $k_o = 0.1$

where

$$\bar{W}_c = \frac{W_c}{\pi p_1 L^2} \ , \ \bar{h}_2 = \frac{h_2}{L}$$

and
$$F_c = \{\frac{1}{2} k^{1-n}(k^2-k_o^2) \frac{k^{3-n}-k_o^{3-n}}{3-n}\} + H^{2n+1} \{\frac{1}{2} (1-k^{1-n})(k^2-k_o^2) + \frac{1}{2}(1-k^2) - \frac{1-k^{3-n}}{3-n}\} \ . \quad (5.26)$$

If $k \to k_o$, then $\bar{W}_c$ is given by

$$\bar{W}_c = \frac{1-n}{3-n} \frac{(1-k_o^{3-n}) \sin^2\alpha}{(1-k_o^{1-n})} + \bar{h}_2(H-1) k_o \sin 2\alpha \ . \quad (5.27)$$

The above result has already been obtained by Shukla [1963-a] when $\bar{h}_s = 0$. Also for $\alpha = 90^\circ$, $\bar{W}_c$ is same as the load capacity of bearing with constant film thickness, Shukla & Prakash [1969]. When $n = 1$ the load capacity can be written as

$$\bar{W}_c = k_o^2 \sin^2\alpha + \frac{2}{g_1} \bar{h}_2 H^3 (H-1) k \ \ln k \sin\alpha \cos\alpha + \frac{F_1 \sin^2\alpha}{g_1} \ , \quad (5.28)$$

where

$$g_1 = H^3 \ \ln k - \ln \frac{k}{k_o}$$

and
$$F_1 = \{k_o^2 \ln \frac{k}{k_o} - \frac{1}{2} (k^2-k_o^2)\} + \frac{H^3}{2} \{(k^2-1) - 2k_o^2 \ln k\} \quad (5.29)$$

The variations of $\bar{W}_c$ with H are shown in figures (5.4) to (5.9). It can be observed from these figures that $\bar{W}_c$ increases as H or h_s increases for fixed k and n, and the load capacity of the step conical bearing is always greater than the corresponding case with uniform film thickness [see equation (5.27)] .

From figures (5.4), (5.5) and (5.6) it can also be pointed out that for fixed H and n, the load capacity can increase or decrease with respect to the increase in k and this variation depends upon the values of H and n. Further from figures (5.7), (5.8) and (5.9) it can be seen that for fixed H and k, $\bar{W}_c$ increases as n decreases for values of k close to k_o but it decreases for larger values of k. Thus it may be concluded that for a given n, $\bar{W}_c$ attains a maximum value for some values of H and k.

5.3 CASE II: NON-CONTACTING HYDROSTATIC STEP SEAL

Consider the case of a non-contacting hydrostatic step seal as shown in figure no. (5.10). A non-Newtonian power law fluid is flowing through the stepped clearance due to high pressure p_i at $x = 0$. Since the seal width L is small in comparision to the outside radius of the seal we can use the infinitely long clearance solution for determining the pressure distribution. From equation (5.13) we can write the equation determining the pressure in this case as follows:

$$\frac{dp_j}{dx} = -m \left\{ \frac{(2n+1)Q}{2nb} \right\}^n \left(\frac{2}{h_j} \right)^{2n+1} , \qquad (5.30)$$

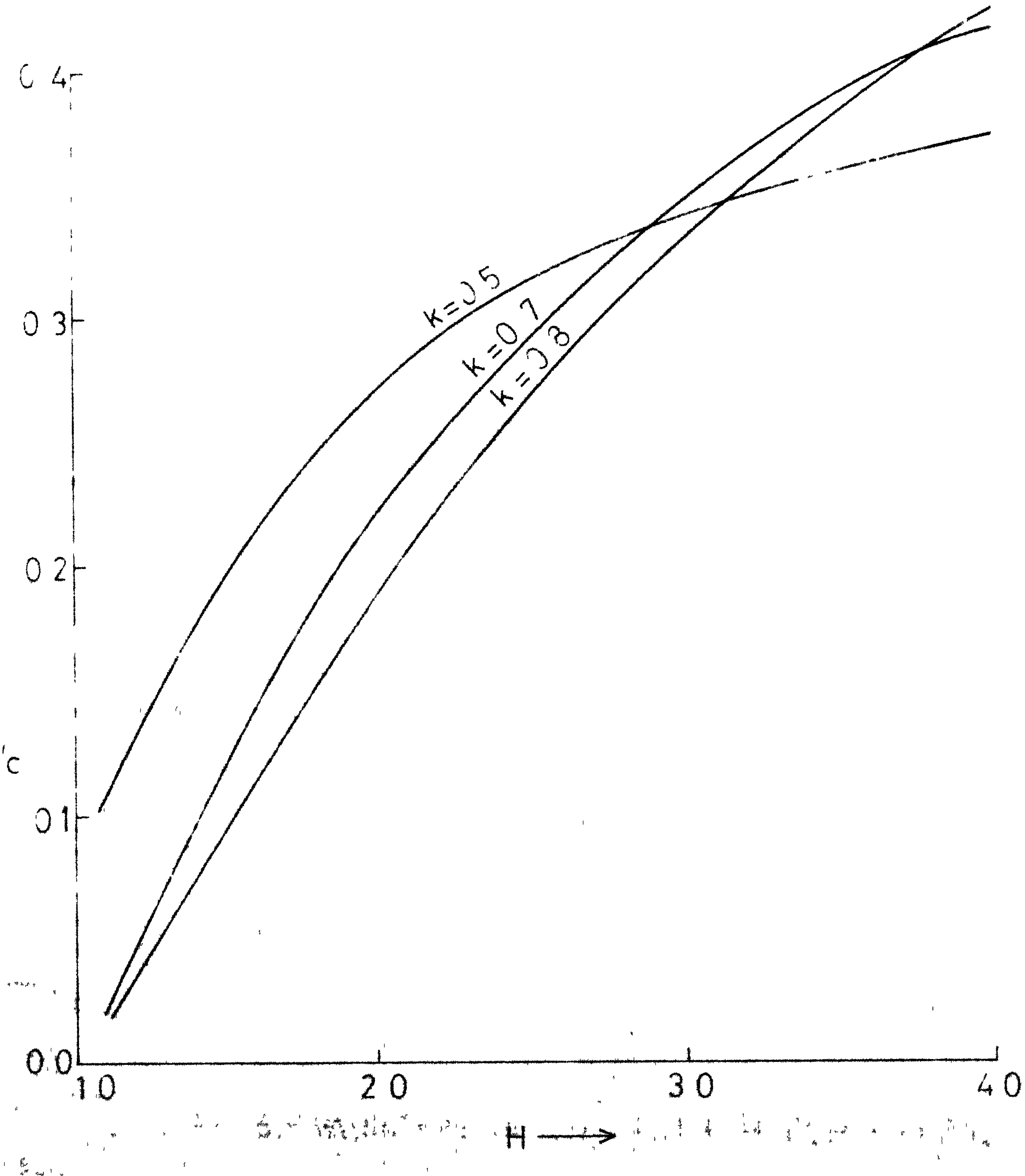


FIG 5.4. VARIATION OF $\overline{W}_c$ WITH H FOR $n=0.5$, $\alpha=60^\circ$, $k_0=0.1$

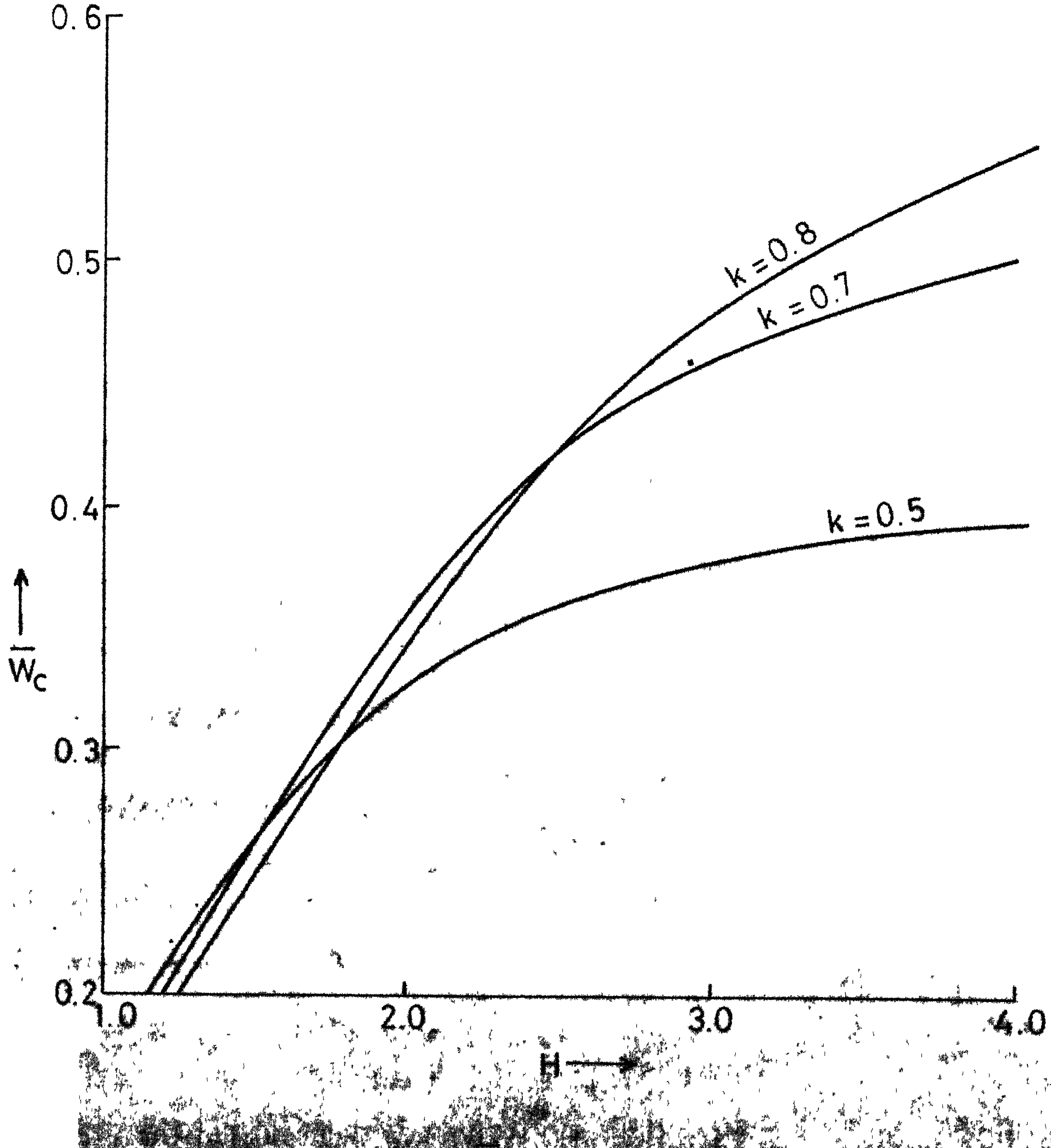


FIG. 5.5 VARIATION OF $\overline{W}_c$ WITH H FOR $n = 1.0$, $\alpha = 60°$, $k_0 = 0.1$

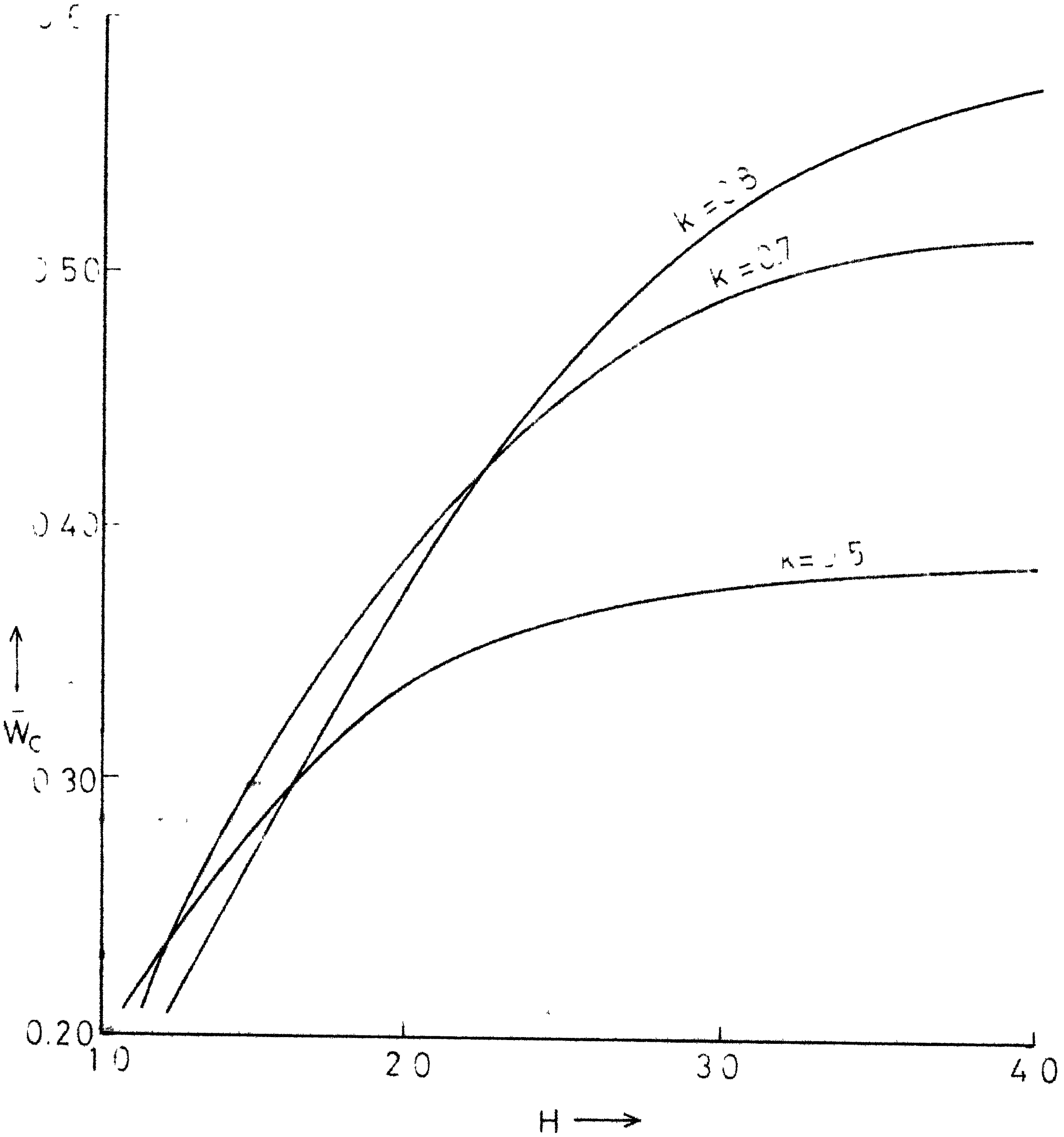


FIG 5.6. VARIATION OF $\overline{W}_c$ WITH H FOR $n=1.5$, $\alpha=60^\circ$, $k_o=0.1$

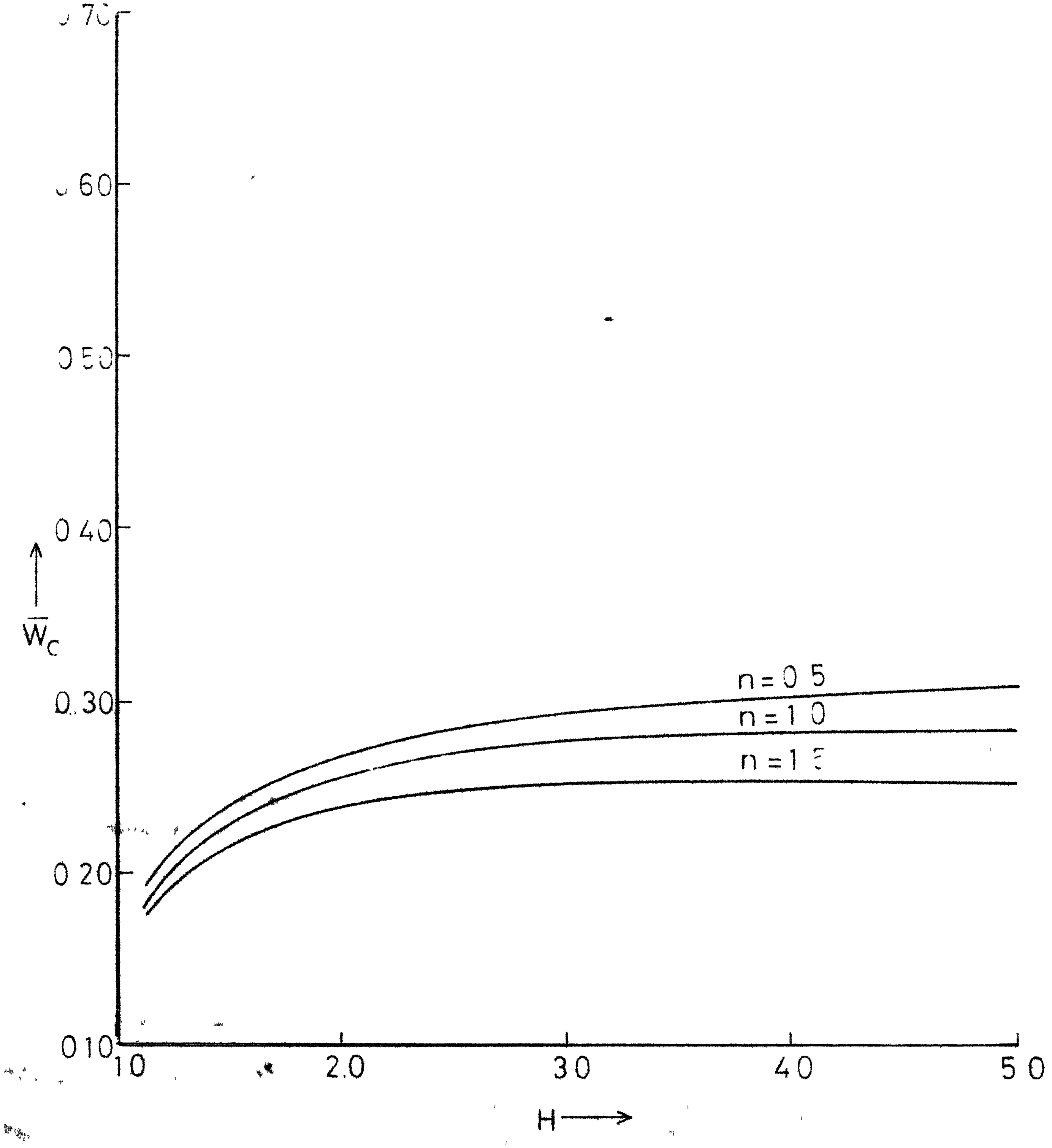


FIG. 5.7 VARIATION OF $\bar{W}_c$ WITH H FOR $k = 0.3$, $\alpha = 60°$, $k_\theta = 0.1$

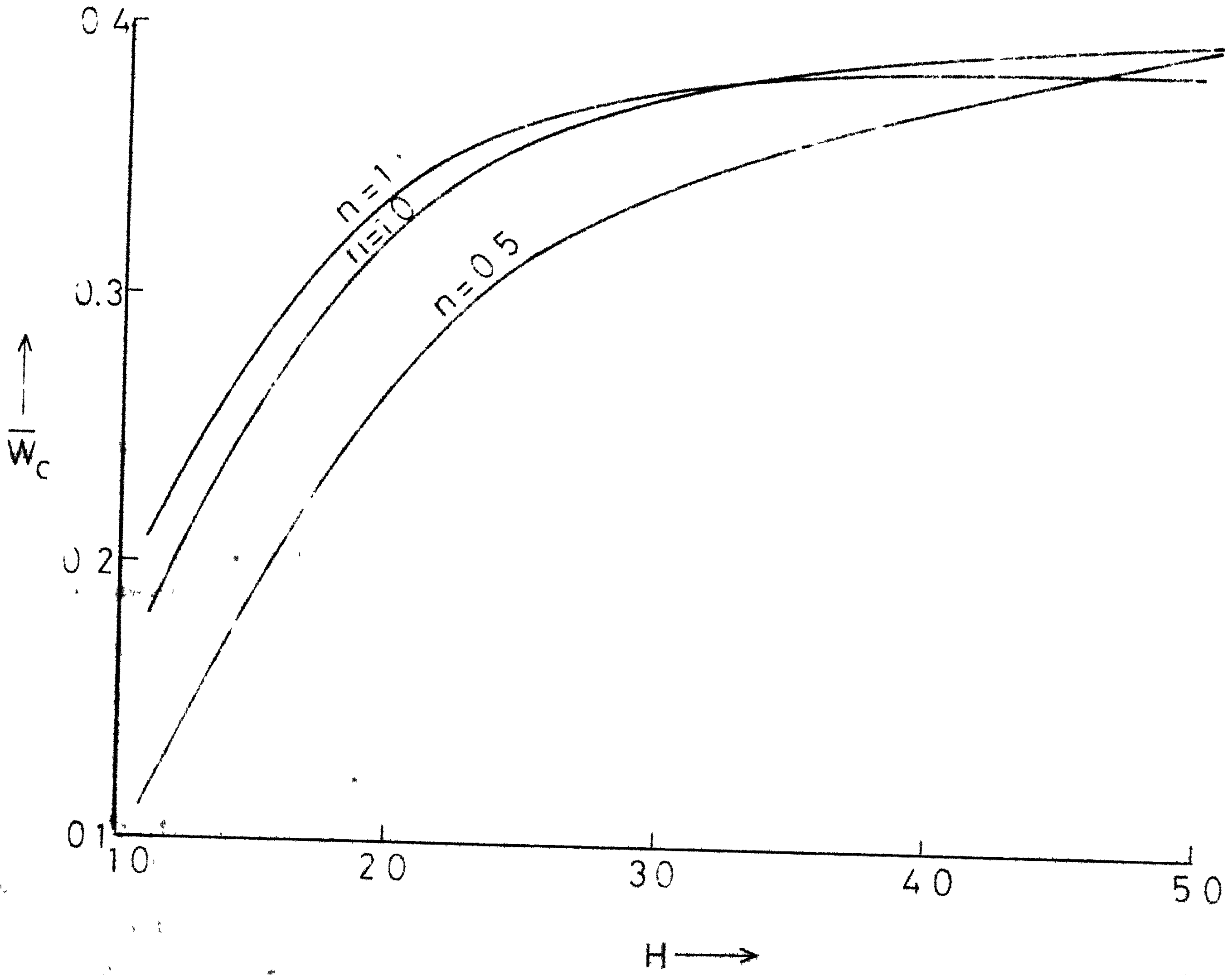


FIG.5.8 VARIATION OF $\overline{W}_c$ WITH H FOR k = 0 5, α = 60°, k_o = 0 1

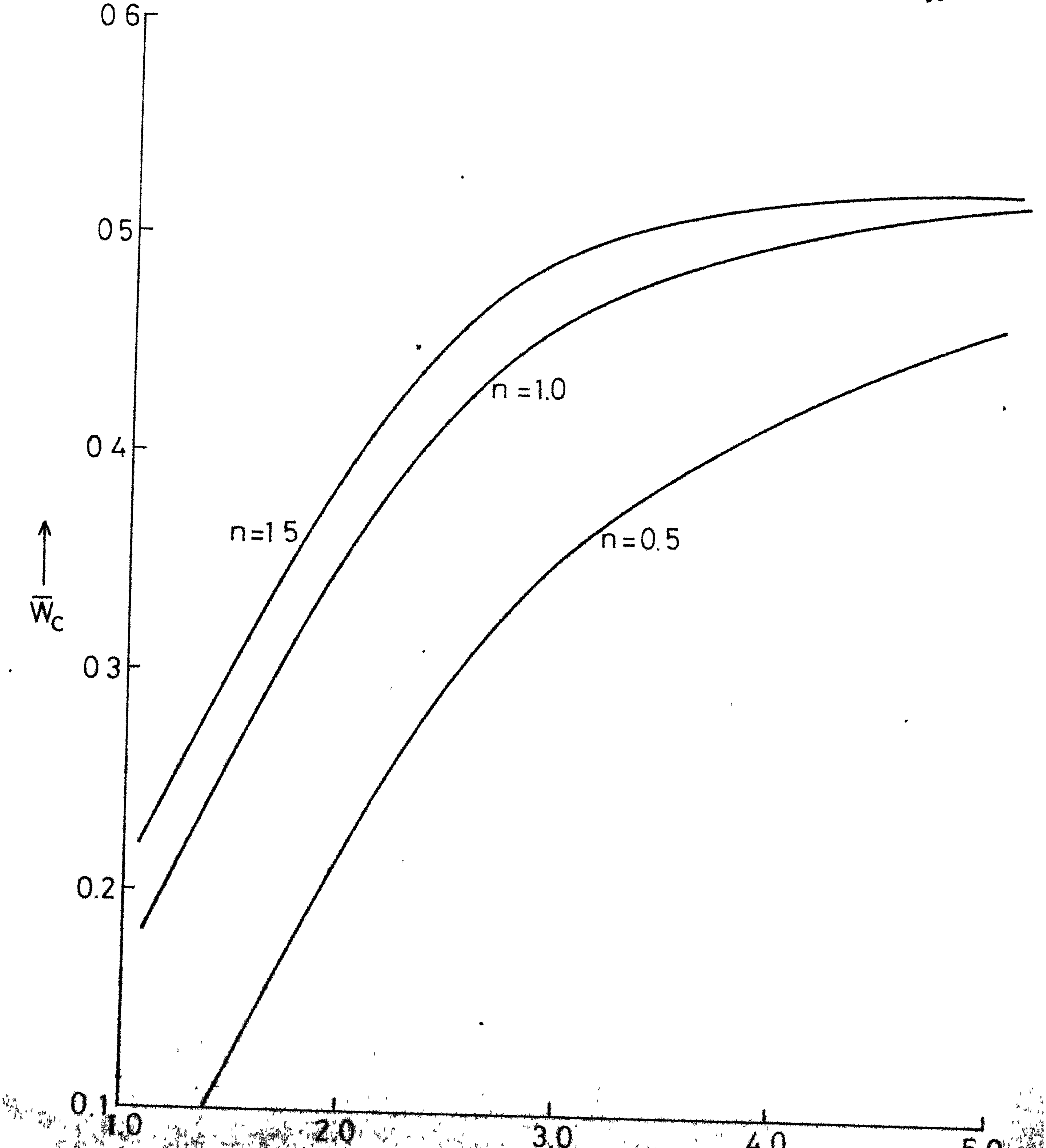


FIG.5.9 VARIATION OF $\overline{W}_c$ WITH H FOR $k = 0.7$, $\alpha = 60°$, $k_0 = 0.1$

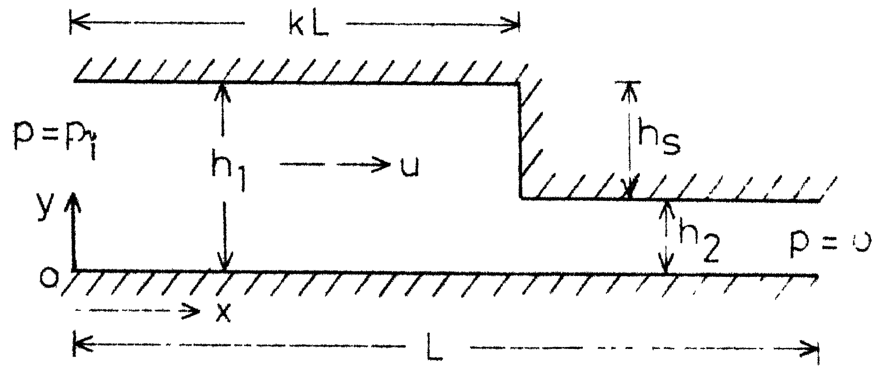


FIG 5.10 HYDROSTATIC NON - CONTACTING STEP SEAL

where $j = 1$ in the region $0 \leq x \leq kL$ and $j = 2$ in the region $kL \leq x \leq L$ and Q is the flow flux which is a constant.
Integrating equation (5.30) and using the boundary conditions

$$\begin{aligned} p_1 &= p_i \quad \text{at} \quad x = 0 \\ p_1 &= p_2 \quad \text{at} \quad x = kL \\ p_2 &= 0 \quad \text{at} \quad x = L \end{aligned} \tag{5.31}$$

we have the following expressions for the pressures in the two regions

$$p_1 = \frac{p_i}{f} [f - (\frac{2}{h_1})^{2n+1} \frac{x}{L}], \; 0 \leq x \leq kL \tag{5.32}$$

$$p_2 = \frac{p_i}{f} (\frac{2}{h_2})^{2n+1} (1 - \frac{x}{L}), \; kL \leq x \leq L, \tag{5.33}$$

where $f = k (\frac{2}{h_1})^{2n+1} + (1-k) (\frac{2}{h_2})^{2n+1}$

and $h_1 = h_2 + h_s$ (5.34)

The expression for the flux can be written as

$$Q = (\frac{p_i}{mLf})^{\frac{1}{n}} \frac{2nb}{2n+1} \; . \tag{5.35}$$

When $h_s = 0$, the flow flux Q_o in the seal is given by

$$Q_o = \frac{2nb}{2n+1} (\frac{p_i}{mL})^{\frac{1}{n}} (\frac{h_2}{2})^{2 + \frac{1}{n}} \; . \tag{5.36}$$

The ratio $\frac{Q_o}{Q}$ is plotted in figure no. (5.11) with H for various values of n. It can be seen from there that this ratio decreases as H or h_s increases and $Q \geq Q_o$ for $H > 1$. Again, since Q_o is not a function of h_s, Q increases as h_s increases for all n.

The load capacity is given by

$$W_s = b \int_0^{kL} p_1 dx + b \int_{kL}^{L} p_2 dx$$

which on using equations (5.32) and (5.33) gives,

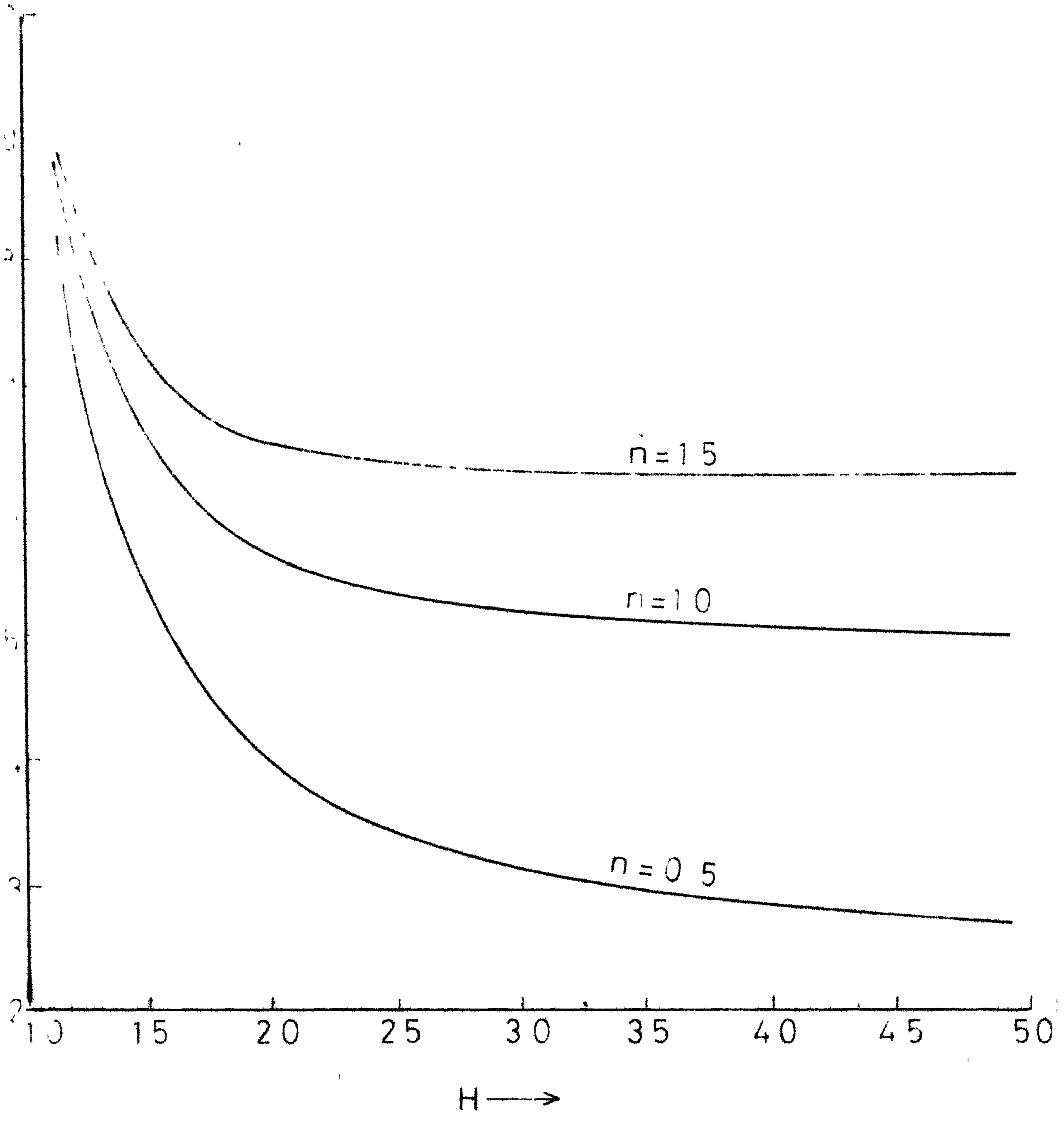
$$\bar{W}_s = \frac{W_s}{bp_i L} = \frac{k^2+(1-k^2)H^{2n+1}}{2[k + (1-k) H^{2n+1}]} . \tag{5.37}$$

From equation (5.37) we have

$$\frac{\partial \bar{W}_s}{\partial (H^{2n+1})} = \frac{k(1 - k)}{2[k + (1-k) H^{2n+1}]^2} > 0 . \tag{5.38}$$

Since $H > 1$ and $n > 0$, H^{2n+1} increases as H or n increases. Thus we observe that $\bar{W}_s$ increases as H or h_s increases for fixed k and n. Further, it can also be concluded that $\bar{W}_s$ increases as n increases for all $H > 1$ and $0 < k < 1$. The variations of $\bar{W}_s$ with H and n for different values of k are shown in figures (5.15) to (5.17). Again, from equation (5.37), we obtain

$$\frac{\partial \bar{W}_s}{\partial k} = \frac{(1-H^{2n+1}) [k^2-(1-k)^2 H^{2n+1}]}{2[k+(1-k)H^{2n+1}]^2} .$$



VARIATION OF Q_0/Q WITH H IN THE CASE OF A STEPSEAL FOR k = 0.5.

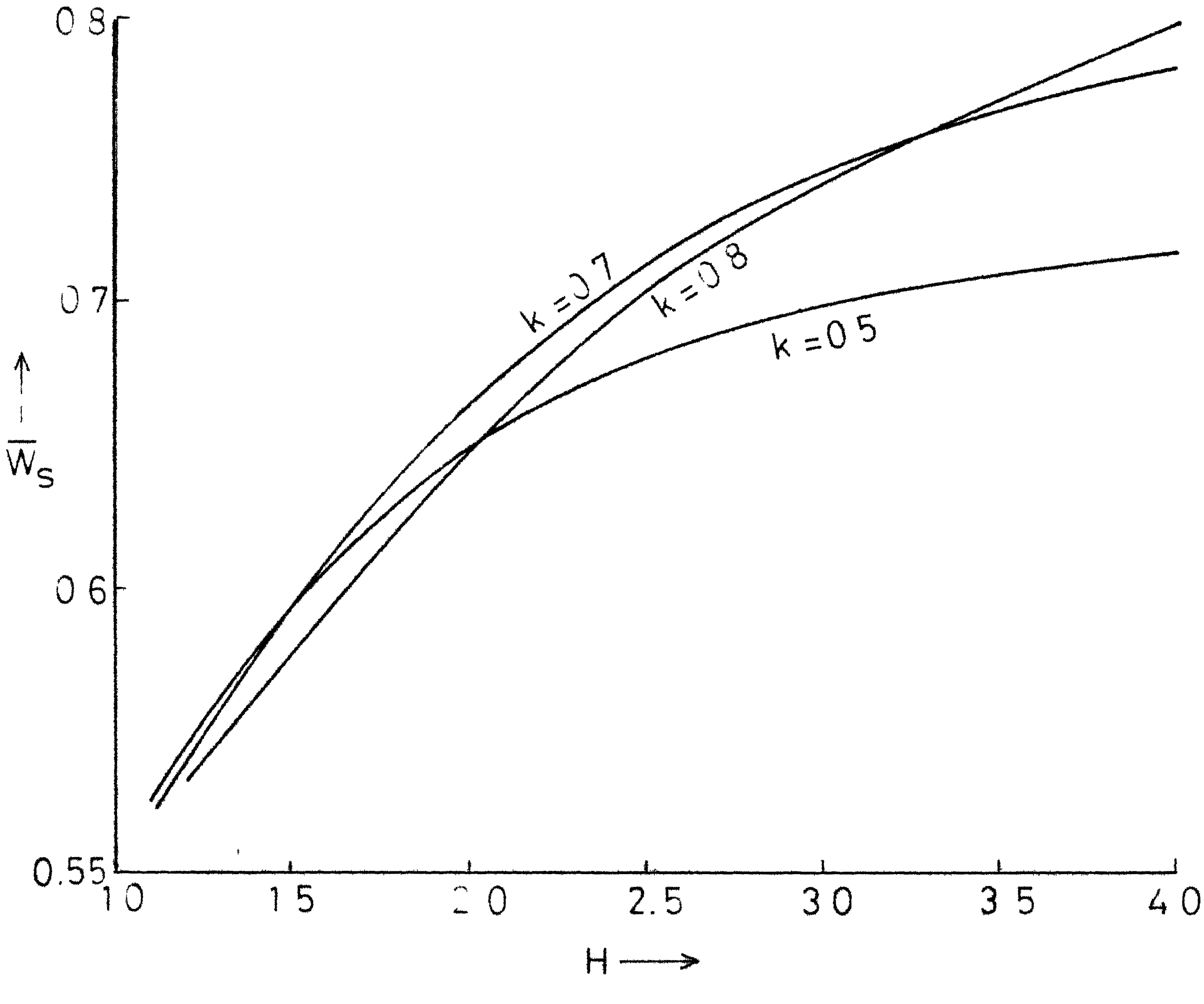


FIG. 5 12 VARIATION OF $\overline{W}_S$ WITH H FOR n = 0 5

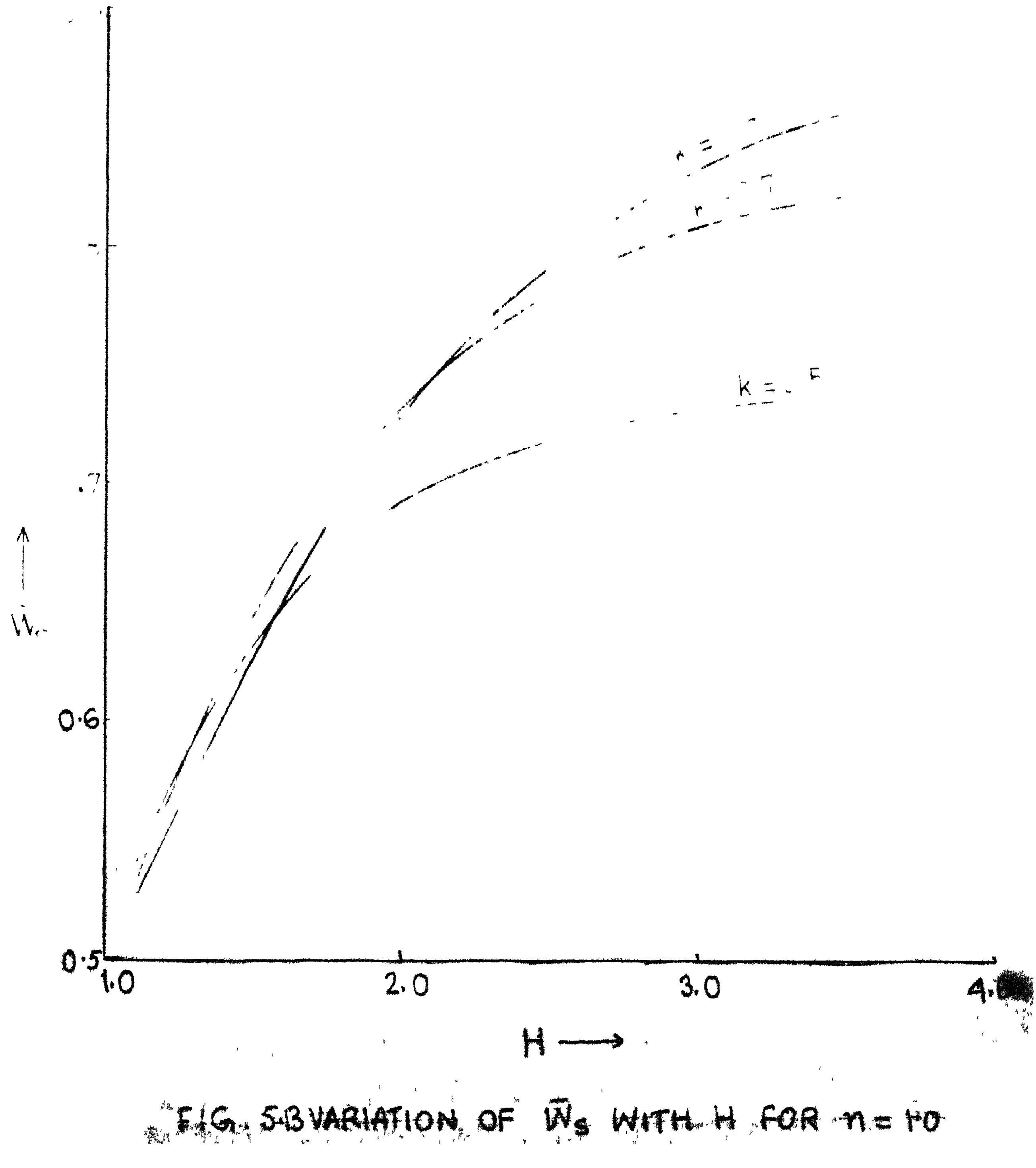


FIG. 5.3 VARIATION OF $\bar{W}_s$ WITH H FOR $n = 1.0$

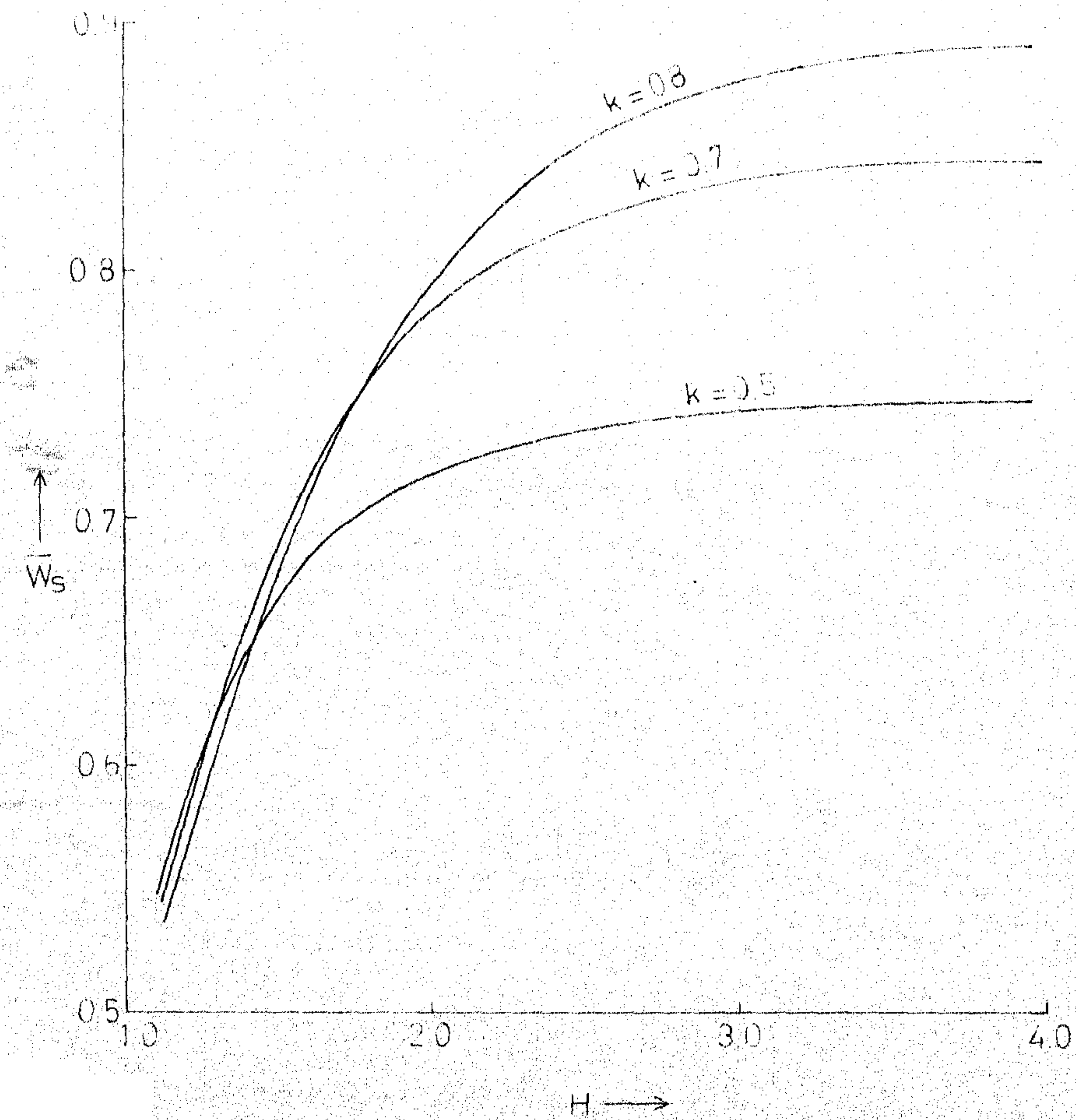


FIG.5.14. VARIATION OF $\overline{W}_s$ WITH H FOR n = 1.5.

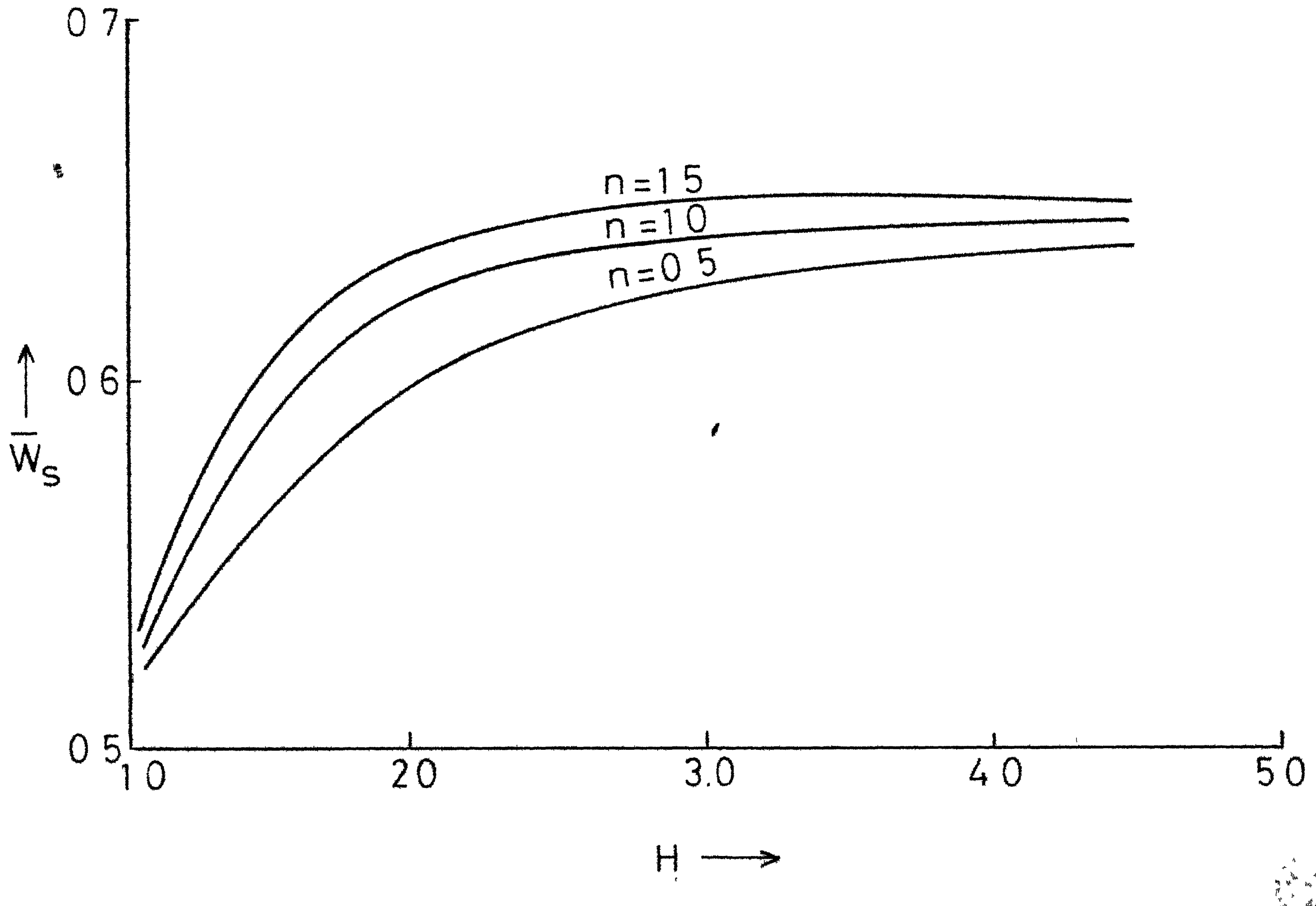


FIG 5 15 VARIATION OF $\overline{W}_s$ WITH H FOR k = 0.3

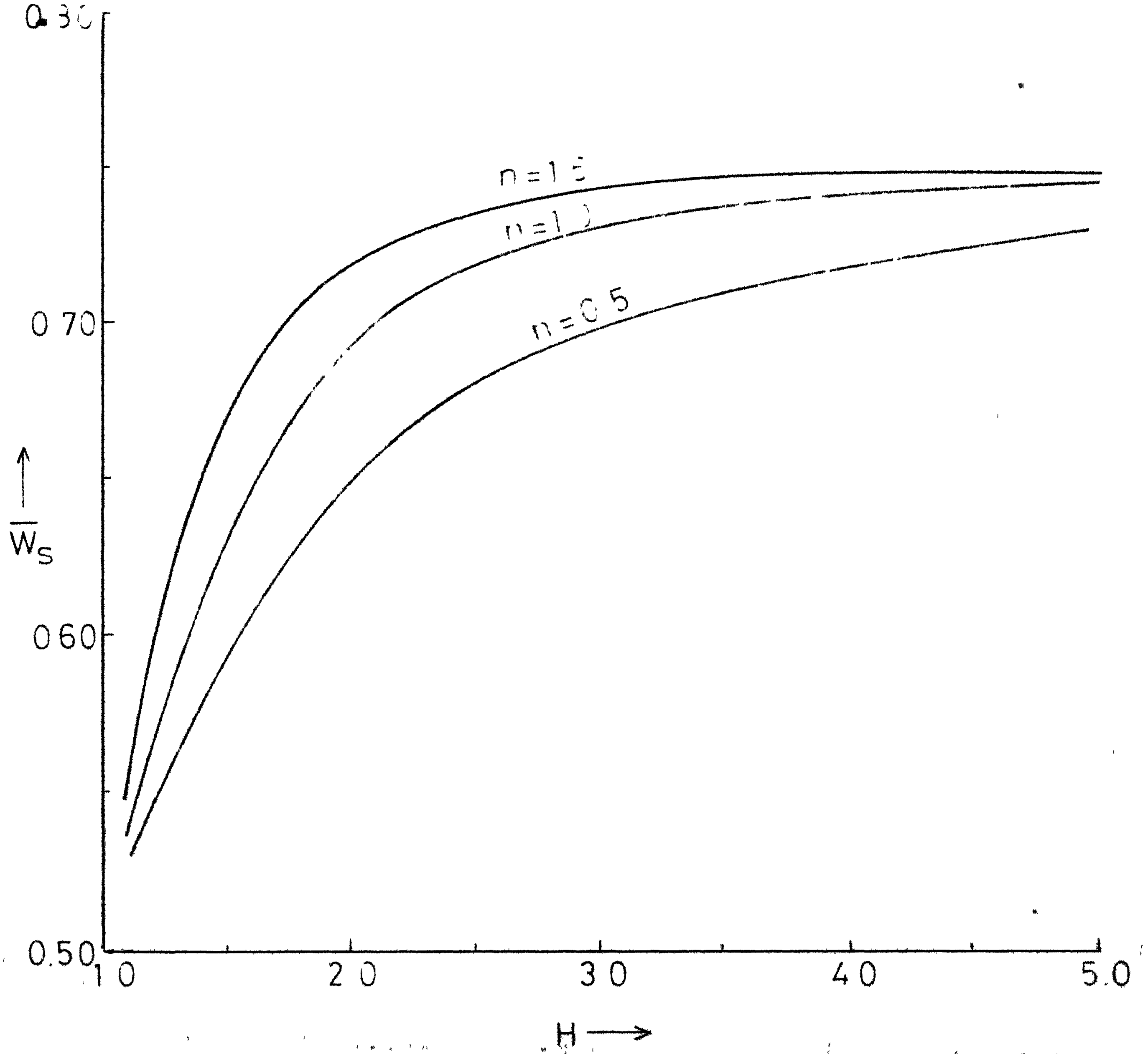


FIG. 5.16. VARIATION OF $\overline{W}_s$ WITH H FOR k = 0.5.

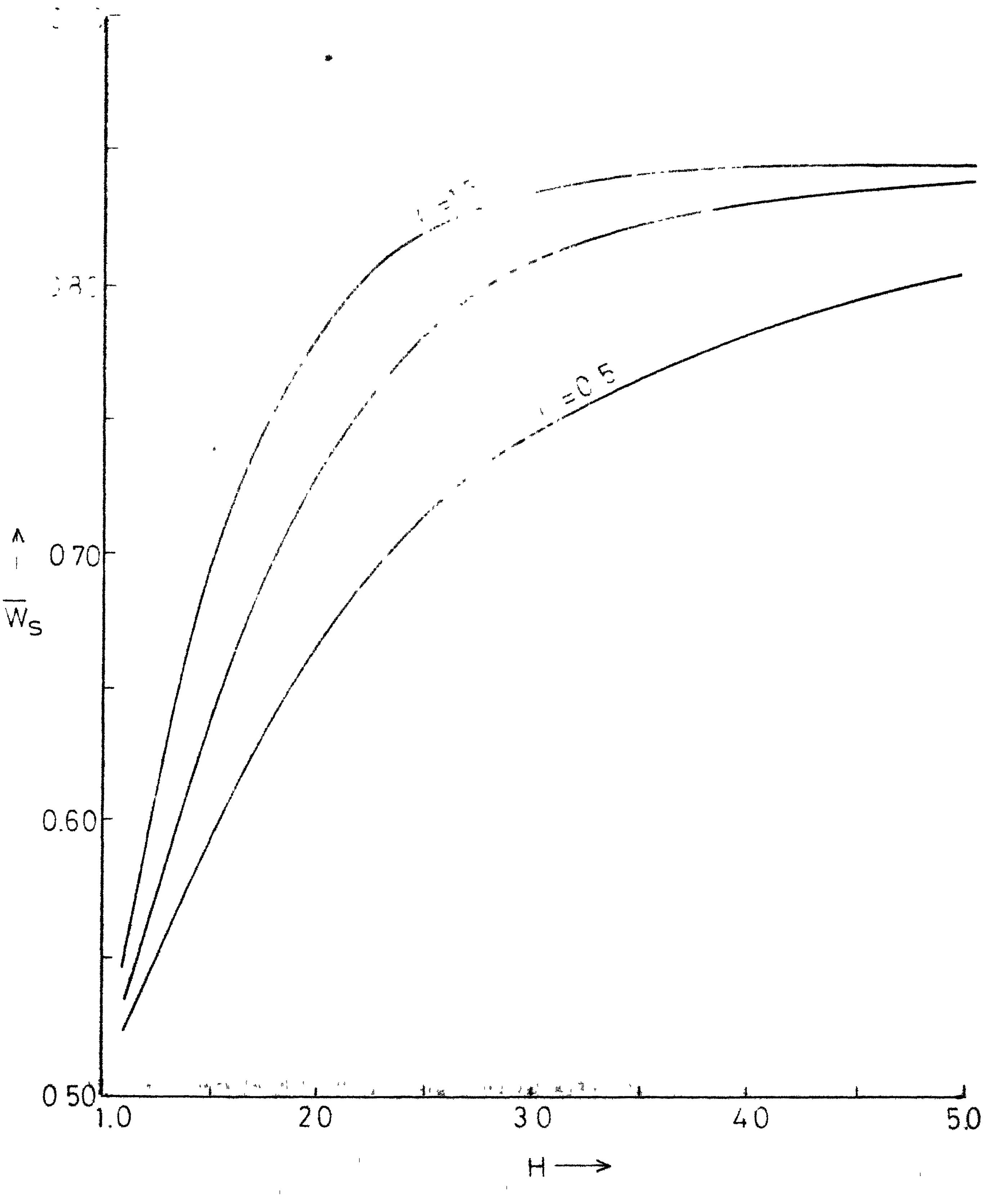


FIG.5 17 VARIATION OF $\overline{W}_S$ WITH H FOR k = 0 7

It is seen that sign of $\frac{\partial \bar{W}_s}{\partial k}$ depends upon H and n for fixed k and is zero for some value of k for given n and H. Hence $\bar{W}_s$ can increase or decrease with increase in k depending upon the values of H and n. The variations of $\bar{W}_s$ with k can be seen from figures (5.12) to (5.14).

If $h_s = 0$ (H = 1, k = 0), from equation (5.37) we have

$$\bar{W}_s = 1/2 \tag{5.39}$$

It is noted from here that the load capacity in case of a seal with uniform gap does not depend on the flow behaviour indices of the lubricant.

5.4 CONCLUSIONS

In this chapter, we have studied the effects of stepped film thickness on the various characteristics of the bearings and seals using power law fluids as lubricants.

The following conclusions may be drawn from the analysis:

(1) In the case of externally pressurised conical bearing the flow flux increases due to increase in the step height. Similar situation occurs in non-contacting hydrostatic step seals.

(2) The load capacity of a conical step bearing is always greater than that of a corresponding bearing with uniform film thickness.

(3) In the case of a conical bearing, for given H and k, the load capacity increases as n decreases for smaller values of k but the situation is reversed for larger values of k. Further, for given H and n, the load capacity may increase or decrease as k increases and this variation depends upon the choice of H and n.

(4) In the case of hydrostatic step seals, the load capacity increases as H or n increases for all values of k. The load capacity may decrease or increase as k increases, depending upon the values of H and n. However, after certain transitional value of H depending upon n, the load capacity may increase with increase in k.

CHAPTER - VI

THERMAL EFFECTS IN EXTERNALLY PRESSURISED BEARING WITH POWER LAW LUBRICANTS

In high temperature and pressure applications the flow behaviour characteristics of the lubricant (Newtonian or non-Newtonian) no longer remain constant, but they vary with temperature and pressure, Rotem and Shinnar [1962] , Turian [1965] , Dyer [1969] . Several investigations have been conducted in recent years to study the characteristics of bearing by considering these variations in the case of Newtonian lubricants, Ting and Mayer [1971] , Rodkiewicz and Anwar [1971] , Donaldson [1971] , Gould [1967; 1971] . In particular, the effect of temperature in externally pressurised bearings has been studied by Donaldson [1971] , Ting and Mayer [1971], using exponential relation between viscosity and temperature.

In this chapter, we study the characteristics of externally pressurised bearings using non-Newtonian power law lubricants where consistency of the lubricant varies exponentially with temperature and pressure.

6.1 BASIC EQUATIONS

As before, let us consider the flow of a non-Newtonian power law lubricant in an infinitely long thin clearance due to external pressurization. The physical situation and the co-ordinate system are shown in figure no. (6.1).

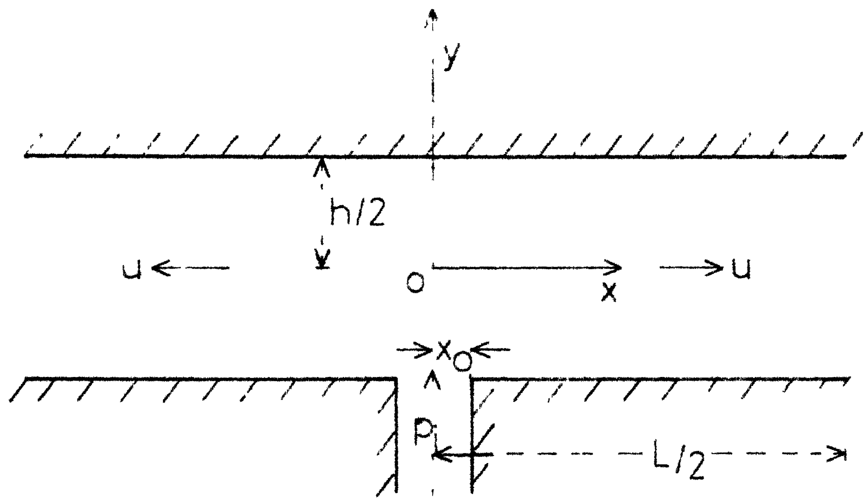


FIG. 6 1 EXTERNALLY PRESSURISED RECTILINEAR PLATES WITH POWER LAW LUBRICANTS

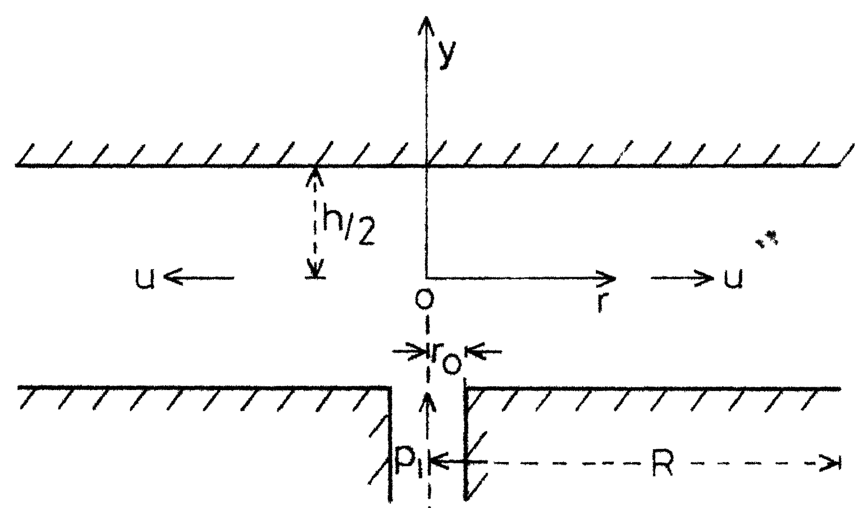


FIG 6 2 EXTERNALLY PRESSURISED CIRCULAR PLATES WITH POWER LAW LUBRICANTS

The equations of momentum and energy which govern the flow of a power law lubricant are given by

$$0 = -\frac{dp}{dx} + \frac{\partial}{\partial y}\{m\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\} \qquad (6.1)$$

$$\rho\, c_v\, u\,\frac{dT}{dx} = k\,\frac{\partial^2 T}{\partial y^2} + \{m\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\}\,\frac{\partial u}{\partial y} \qquad (6.2)$$

where m, the consistency index, varies exponentially with temperature and pressure as:

$$m = m_o\, e^{\alpha p - \beta T} \qquad (6.3)$$

Since the physical configuration is symmetric, we consider the region $0 \leq y \leq h/2$ for further investigation. In the region $0 \leq y \leq h/2$, clearly $\frac{\partial u}{\partial y} \leq 0$ and equation (6.1) can be written as

$$\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n = -\frac{1}{m}\frac{dp}{dx}\ . \qquad (6.4)$$

Integrating equation (6.4) and using the boundary condition

$$\begin{aligned} \frac{\partial u}{\partial y} &= 0 \quad \text{at} \quad y = 0 \\ u &= 0 \quad \text{at} \quad y = h/2\ , \end{aligned} \qquad (6.5)$$

we get

$$\frac{\partial u}{\partial y} = -\left(-\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} y^{\frac{1}{n}}\ , \qquad (6.6)$$

$$u = \left(-\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} \frac{\left(\frac{h}{2}\right)^{1+\frac{1}{n}} - y^{1+\frac{1}{n}}}{1+\frac{1}{n}} \tag{6.7}$$

The volume flux Q of the lubricant is defined by

$$Q = b\ \{\int_{-h/2}^{0} udy + \int_{0}^{h/2} udy\}$$

which on account of symmetry reduces to

$$Q = 2b \int_{0}^{h/2} udy\ . \tag{6.8}$$

Making use of equation (6.7) in equation (6.8), we obtain the final expression for flow flux as

$$Q = \frac{2nb}{2n+1}\left(-\frac{1}{m}\ \frac{dp}{dx}\right)^{\frac{1}{n}}\left(\frac{h}{2}\right)^{2+\frac{1}{n}}\ . \tag{6.9}$$

It can be easily seen from the equation of continuity that the flow flux remains constant and does not depend on x. In order to consider the effect of temperature we write the equation (6.2) in the following form:

$$\rho c_v\ u\ \frac{dT}{dx} = k\ \frac{\partial^2 T}{\partial y^2} + m\left(-\ \frac{\partial u}{\partial y}\right)^{n+1}\ ,\quad 0 \le y \le h/2\ . \tag{6.10}$$

Averaging the equation (6.10) from y = 0 to y = h/2 and making use of the symmetry we get

$$\rho c_v \frac{h}{2} \frac{dT}{dx} = 2n \int_0^{h/2} \left(- \frac{\partial u}{\partial y}\right)^{r+1} dy \tag{6.11}$$

where $k \frac{\partial T}{\partial y} \Big|_{-h/2}^{+h/2}$ is neglected.

From equations (6.6), (6.9) and (6.11), we finally obtain,

$$\rho c_v \frac{dT}{dx} = - \frac{dp}{dx} \ . \tag{6.12}$$

The boundary conditions for T and p are

$$\begin{aligned} T &= 0 \quad \text{at} \quad x = x_o \\ p &= p_i \quad \text{at} \quad x = x_o \end{aligned} \tag{6.13}$$

Integrating equation (6.12) and using the conditions (6.13) we get

$$T = - \frac{1}{\gamma} (p - p_i) \quad , \tag{6.14}$$

where $\gamma = \rho c_v$

From equations (6.3) and (6.14) we get the final expression for consistency index as

$$m = m_o \, e^{\frac{-\beta p_i}{\gamma}} \, e^{\frac{\lambda T}{p_i}} \quad , \tag{6.15}$$

where $\lambda = (\alpha + \frac{\beta}{\gamma}) p_i$ is a dimensionless parameter.

Thus the dependence of consistency on temperature and pressure ultimately reduces to pressure dependence in this case.

6.2 EXTERNALLY PRESSURISED INFINITELY EXTENDED RECTANGULAR THRUST PLATES

In this case, from equations (6.9) and (6.15) the final expression for determining the pressure of the lubricant can be written as:

$$\frac{2nh}{2n+1}\left(-\frac{1}{m_o}\, e^{\frac{\beta p_i}{\gamma}}\, e^{\frac{-\lambda p}{p_i}}\, \frac{dp}{dx}\right)^{1/n} \left(\frac{h}{2}\right)^{2+\frac{1}{n}} = Q = a \text{ constant} . \qquad (6.16)$$

The boundary conditions for p are

$$\begin{aligned} p &= p_i \quad \text{at } x = x_o , \\ p &= 0 \quad \text{at } x = \frac{L}{2} . \end{aligned} \qquad (6.17)$$

Integrating equation (6.16) and apply'ing (6.17), the final expression for p is given by

$$p = -\frac{p_i}{\lambda}\, \ell n \left[1 - \left(\frac{1-2\bar{x}}{1-2\bar{x}_o}\right)(1 - e^{-\lambda})\right] , \qquad (6.18)$$

where $\bar{x} = \frac{x}{L}$, $\bar{x}_o = \frac{x_o}{L}$.

From equations (6.16) and (6.18), the expression for the flow flux can now be written as

(6.19)

$$Q = \frac{2nb}{2n+1}\left[\frac{2p_i(1 - e^{-\lambda})\, e^{\frac{\beta p_i}{\gamma}}}{m_o\, L\, \lambda(1-2\bar{x}_o)}\right]^{\frac{1}{n}} \left(\frac{h}{2}\right)^{2 + \frac{1}{n}} \tag{6.19}$$

The flow flux Q_o for constant consistency index is obtained from equation (6.19) by making λ , $\beta \to 0$ as follows:

$$Q_o = \frac{2nb}{2n+1}\left[\frac{2p_i}{m_o L(1-2\bar{x}_o)}\right]^{\frac{1}{n}} \left(\frac{h}{2}\right)^{2 + \frac{1}{n}} \tag{6.20}$$

From equations (6.19) and (6.20) we get

$$\frac{Q}{Q_o} = \left[\frac{e^{\frac{\beta p_i}{\gamma}} (1 - e^{-\lambda})}{\lambda}\right]^{\frac{1}{n}} \tag{6.21}$$

The load capacity of the bearing is defined as

$$W_F = 2b \int_0^{x_o} p_i dx + 2b \int_{x_o}^{L/2} pdx \tag{6.22}$$

which on using equation (6.18), gives

$$\bar{W}_F = \frac{W_F}{b\, p_i L} = \frac{1 - (1 + \lambda)e^{-\lambda} - 2\bar{x}_o(1 - \lambda - e^{-\lambda})}{\lambda(1 - e^{-\lambda})} \tag{6.23}$$

It is interesting to note from equations(6.19) and (6.23) that for prescribed applied pressure the flow flux depends upon the flow behaviour characteristics of the lubricant while the load capacity is independent of these indices.

6.3 EXTERNALLY PRESSURISED CIRCULAR BEARING

In this section, we consider the case of externally pressurised bearing, the physical configuration of which is illustrated in figure no. (6.2). In this case, from equations (6.9) and (6.15) by putting $b = 2\pi r$, the final expression for determining the pressure of the lubricant is given by

$$\frac{4n\pi r}{2n+1}\left(-\frac{1}{m_o} e^{\frac{\beta p_i}{\gamma}} e^{-\frac{\lambda p}{p_i}} \frac{dp}{dr}\right)^{\frac{1}{n}} \left(\frac{h}{2}\right)^{2+\frac{1}{n}} = Q = \text{a constant} \qquad (6.24)$$

The boundary conditions for p are

$$\left.\begin{aligned} p &= p_i \quad \text{at} \quad r = r_o \,, \\ p &= 0 \quad \text{at} \quad r = R \end{aligned}\right\} \qquad (6.25)$$

Integrating equation (6.24) and using the condition (6.25) we get the expression for p as

$$p = -\frac{p_i}{\lambda}\,\ln\left[1 - (1 - e^{-\lambda})\,\frac{1 - \bar{r}^{\,1-n}}{1 - \bar{r}_o^{\,1-n}}\right], \qquad (6.26)$$

where $\bar{r} = \frac{r}{R}$, $\bar{r}_o = \frac{r_o}{R}$

From equations (6.24) and (6.26) the expression for the flow flux Q is given by

$$Q = \frac{4n\pi}{2n+1}\left(\frac{h}{2}\right)^{2+\frac{1}{n}}\left[\frac{e^{\frac{\beta p_i}{\gamma}}\, p_i (1-n)(1 - e^{-\lambda})}{\lambda\, m_o (R^{1-n} - r_o^{1-n})}\right]^{\frac{1}{n}} \qquad (6.27)$$

Here also the flux Q_o corresponding to the constant consistency index is obtained by letting β and $\lambda \to 0$, as follows

$$Q_o = \frac{4n\pi}{2n+1}\left(\frac{h}{2}\right)^{2+\frac{1}{n}}\left[\frac{p_i(1-n)}{m_o(R^{1-n}-r_o^{1-n})}\right]^{\frac{1}{n}} \tag{6.28}$$

From equations (6.27) and (6.28) we have

$$\frac{Q}{Q_o} = \left[e^{\frac{\beta p_i}{\gamma}}\,\frac{1-e^{-\lambda}}{\lambda}\right]^{\frac{1}{n}} \tag{6.29}$$

The load capacity of the bearing is defined as

$$W = \pi p_i r_o^2 + 2\pi\int_{r_o}^{R} p r\, dr \tag{6.30}$$

which on using equation (6.25) gives

$$\bar{W} = \frac{W}{\pi p_i R^2} = -\int_{\bar{r}_o}^{1} \bar{r}^2 \frac{dp}{d\bar{r}}\, d\bar{r} \tag{6.31}$$

Substituting for $\frac{dp}{d\bar{r}}$ from equation (6.26), the final expression for load capacity is given by

$$\bar{W} = \frac{(1-e^{-\lambda})(1-n)}{\lambda}\int_{\bar{r}_o}^{1} \frac{\bar{r}^{2-n}}{(1-\bar{r}_o^{1-n})-(1-e^{-\lambda})(1-\bar{r}^{1-n})}\, d\bar{r} \tag{6.32}$$

The expression for the load capacity $\bar{W}_1$ for Newtonian lubricant is obtained from equation (6.32) by letting $n \to 1$, which gives

$$\bar W_1 = \frac{1 - e^{\lambda}}{\lambda} \int_{r_o}^{1} \frac{\bar r \, d\bar r}{(1 - e^{-\lambda}) \ln \bar r - \ln r_o} \qquad (6\ 33)$$

In the case of constant consistency index the equations (6 32) and (6 33) simplify to (by making $\lambda \to 0$)

$$\bar W = \frac{1-n}{3-n} \frac{1-\bar r_o^{\,3-n}}{1-\bar r_o^{\,1-n}} , \quad n \neq 1 \qquad (6\ 34)$$

$$\bar W_1 = \frac{1-\bar r_o^{\,2}}{2\ln(\frac{1}{\bar r_o})} , \quad n = 1 \qquad (6\ 35)$$

It is easily seen that equation (6 34) is the same as equation (5 27) with $\bar h_s = 0$, while equation (6 35) is the well known result for hydrostatic thrust bearing, Pinkus [1960]

The expressions for load capacity for large λ can be written from equations (6 32) and (6 33) as:

$$\bar W' = \frac{1}{\lambda} \int_{r_o}^{1} \frac{(1-n)\bar r^{\,2-n}}{\bar r^{\,1-n} - \bar r_o^{\,1-n}} d\bar r , \quad n \neq 1$$

$$(6\ 36)$$

$$\bar W_1' = \frac{1}{\lambda} \int_{r_o}^{1} \frac{\bar r}{\ln \frac{\bar r}{\bar r_o}} d\bar r , \quad n = 1$$

It is easily seen from these equations that for large λ the load capacity decreases as λ increases for all n

6 4 DISCUSSION AND RESULTS

It is seen from equations (6 21) and (6 29) that the expressions for Q/Q_o in both the cases of rectilinear and circular plates are the same The variations of Q/Q_o are shown in figures (6 3) and (6 4) for different n It is noted that Q/Q_o increases as $\frac{\beta p_1}{\gamma}$ increases for fixed αp_i It can be further seen from these figures that the ratio Q/Q_o can increase or decrease as n increases and this change depends upon the choice of αp_i and $\frac{\beta p_1}{\gamma}$

The variations of $\bar{W}_p$ in the case of rectilinear externally pressurised thrust bearing are shown in figure no (6 6) for two values of recess It is noted from the expression for $\bar{W}_p$ from equation (6 23) that the load capacity is independent of the flow behaviour indices of the lubricant Further, it may be pointed out that $\bar{W}_I$ decreases as λ increases The variations of the dimensionless load capacity in the case of externally pressurised circular plates are manifested in figure no (6 5) It may be mentioned that $\bar{W}$ decreases as λ or n increases

By putting $n = 1$ and $x_o = 0$ or $r_o = 0$, we get the results obtained by Donaldson [1971] Further, when $\lambda \to 0$, we get the corresponding result of non-Newtonian power law lubricant without thermal effects [see equation (6 34)] , [Shukla 1963-a]

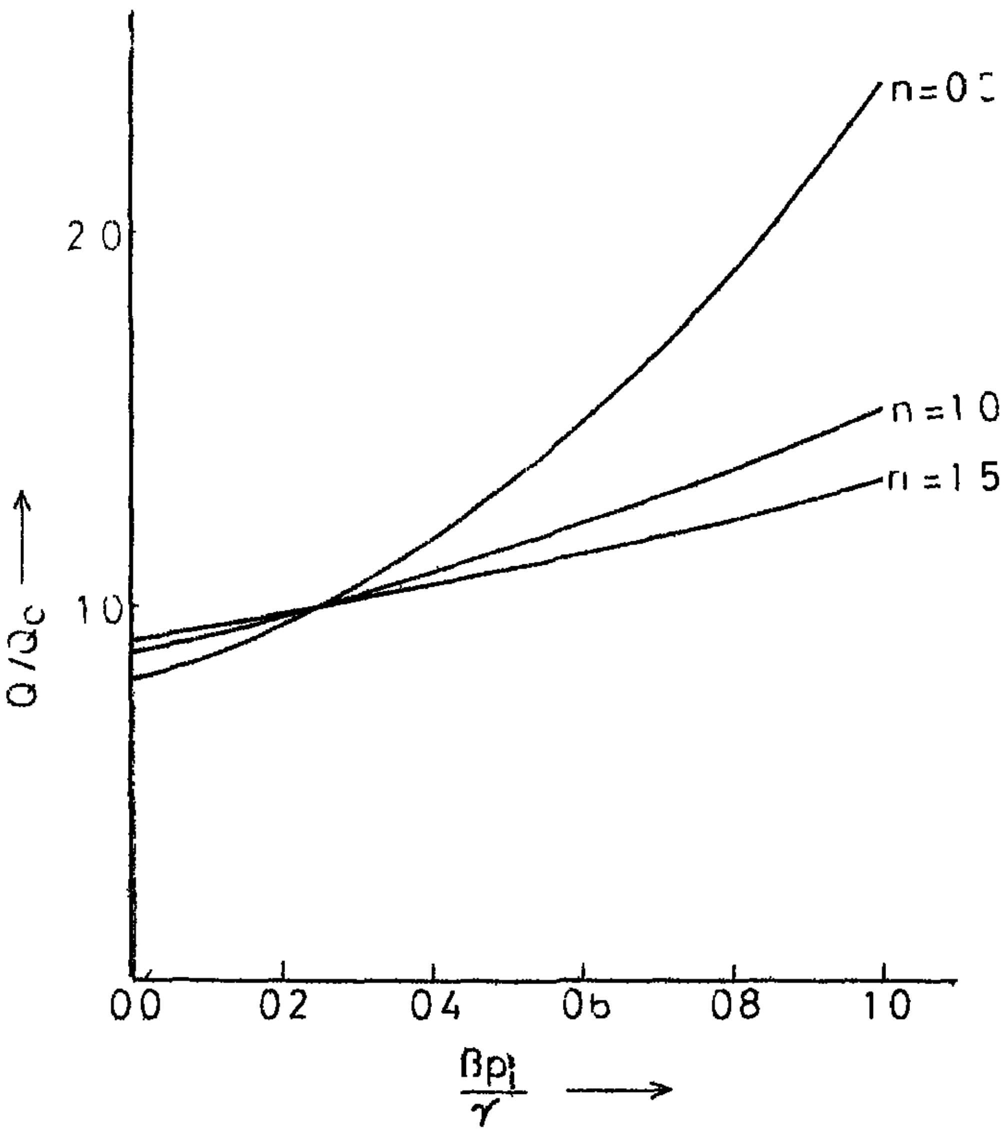


FIG 6 3 VARIATION OF Q/Q_0 WITH $\frac{\beta p_1}{\gamma}$ FOR $\alpha p_1 = 0\ 25$

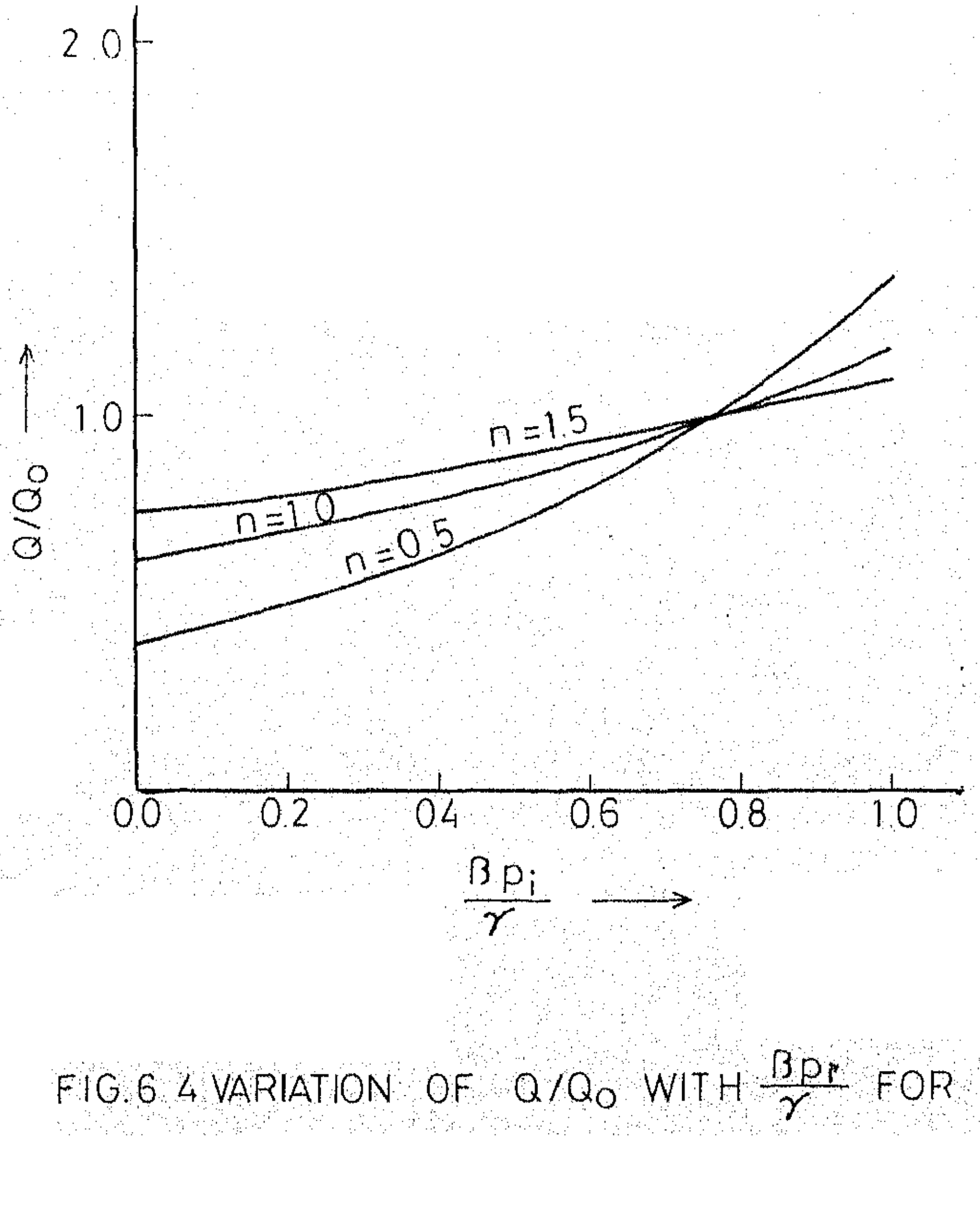


FIG. 6.4 VARIATION OF Q/Q_o WITH $\frac{\beta p_i}{\gamma}$ FOR $\alpha p_i = 1.0$

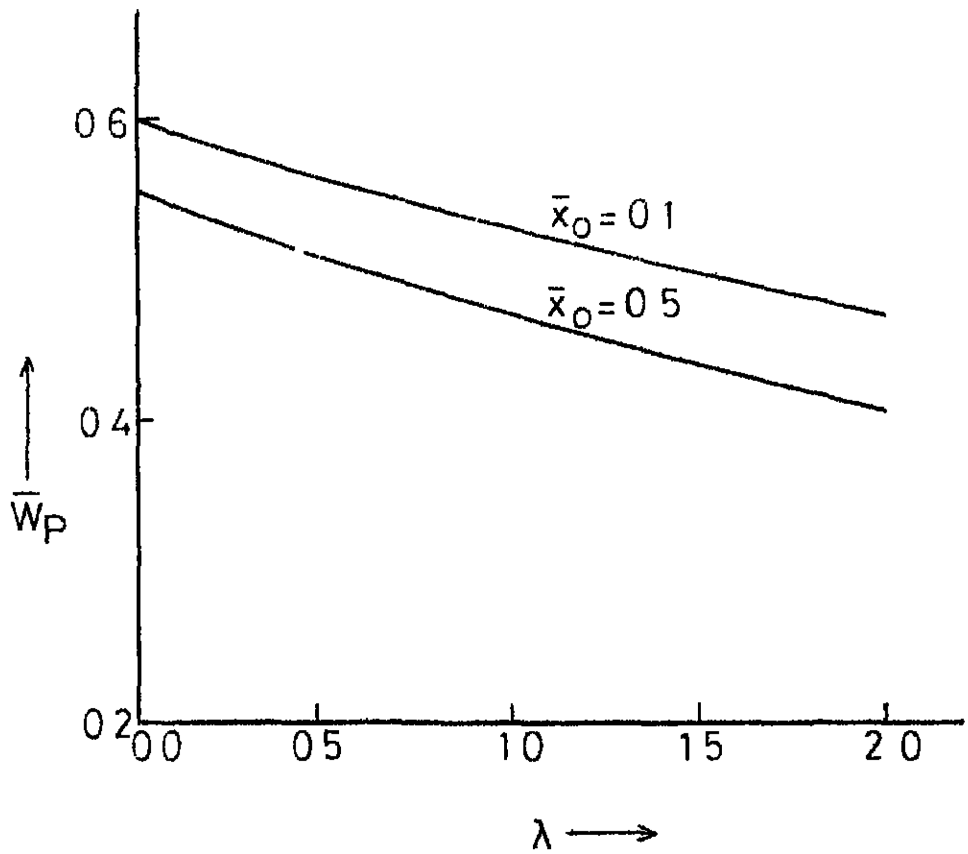


FIG 6 5 VARIATION OF $\bar{W}_P$ WITH λ IN CASE OF EXTERNALLY PRESSURISED RECTILINEAR PLATES FOR DIFFERENT $\bar{x}_o$

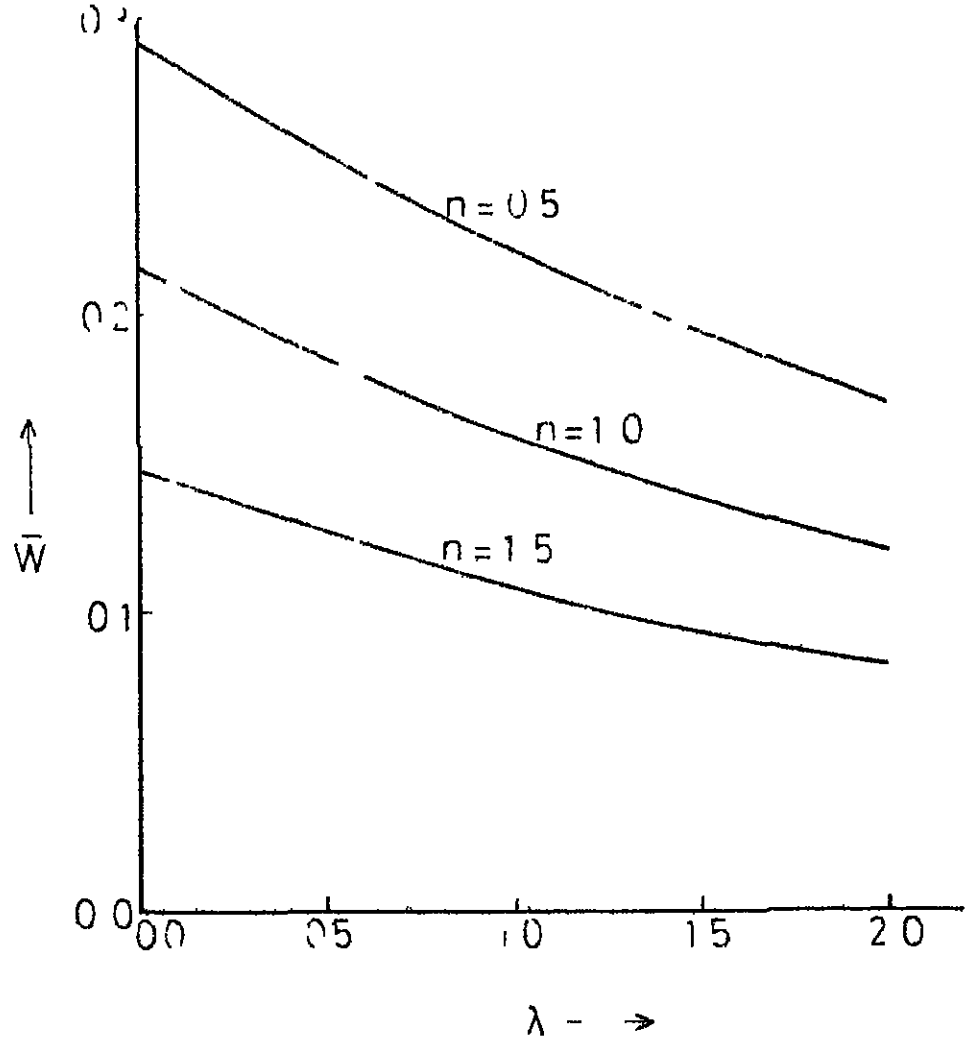


FIG 6.6 VARIATION OF $\bar{W}$ WITH λ IN CASE OF EXTERNALLY PRESSURISED CIRCULAR PLATES WITH $\bar{r}_0 = 0.1$

CHAPTER - VII

EXTERNALLY PRESSURISED POROUS THRUST BEARING WITH POWER LAW LUBRICANTS

The characteristics of externally pressurised porous bearings using Newtonian fluids as lubricants have been studied recently due to their much wider applications, Mori, et al [1965] , Hsing [1971] As most of the lubricants are polymer solutions, the question naturally arises as to how the characteristics of bearings change when such rheological substances known as non-Newtonian fluids are used as lubricants This and other related questions have been answered by many authors who have studied the characteristics of various non-porous bearings by considering Power law models of the lubricants, Ng and Saibel [1962] , Tanner [1963] , Shukla [1963a] , Hsu and Saibel [1965] , Shukla and Prakash [1969] In particular, the characteristics of power law fluid as lubricant have been studied in externally pressurised conical , Shukla [1963a] and circular thrust bearings, Shukla and Prakash [1969] , without taking perosity into account

In this chapter, we study the characteristics of externally pressurised porous thrust bearing using power law fluids as lubricants Here we have assumed both the plates to be porous to exploit symmetry in calculations, however, in actual applications only one of the plates needs to be porous and the results presented here would be equally applicable in this case The analysis is based on the assumption

that the porous matrix consists of a system of capillaries of very small radii and thus restricting the flow of the lubricant through the matrix in only one direction, Hsing [1971] As the exact analytical solution of the problem is not possible, both approximate and numerical solutions are presented The physical situation of the system considered is shown in figure no (7 1)

7 1 BASIC EQUATIONS

The basic equation governing the flow of the lubricant between the bearing surfaces, under usual lubrication assumptions (as in §5 1) is given by

$$\frac{\partial}{\partial y}\{\tau_{xy}\} = \frac{dp}{dx} \quad , \tag{7 1}$$

where $\tau_{xy} = m \left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}$

Since, in the region $H \leq y \leq H + \frac{h}{2}$, $\frac{\partial u}{\partial y} \geq 0$, from equation (7,1), we have

$$\frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y}\right)^n = \frac{1}{m} \frac{dp}{dx} \tag{7 2}$$

Integrating equation (7 2) with boundary conditions $\frac{\partial u}{\partial y} = 0$ at $y = H + \frac{h}{2}$, $u = 0$ at $y = H$, we get

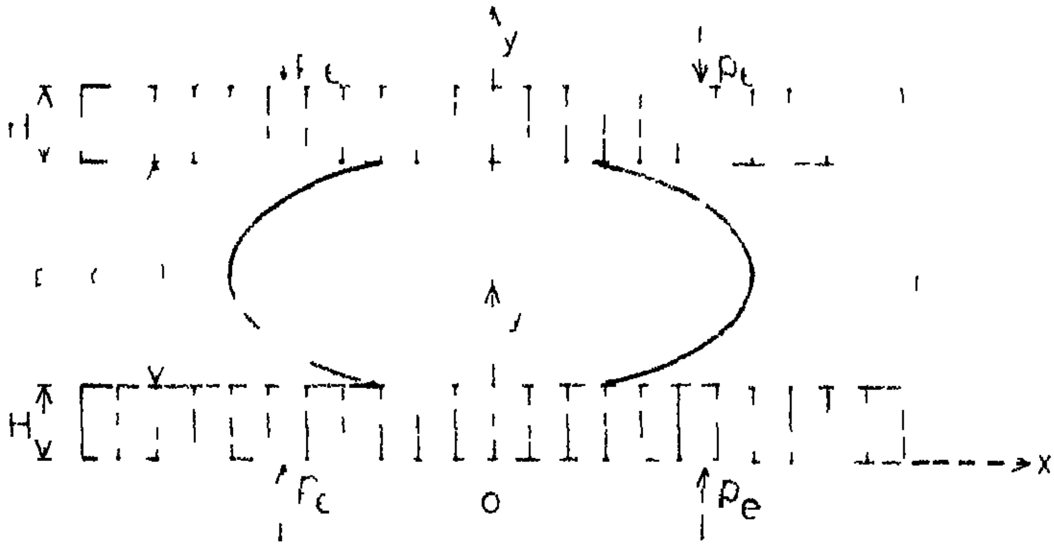


FIG 7.1 FLOW OF A POWER LAW FLUID IN A THIN CLEARANCE BETWEEN TWO EXTERNALLY PRESSURISED PARALLEL POROUS PLATES

$$u = \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \frac{\left(\frac{h}{2} \right)^{1+\frac{1}{n}} - \left(H + \frac{h}{2} - y \right)^{1+\frac{1}{n}}}{1+\frac{1}{n}} \qquad (7\ 3)$$

Similarly, in the region $H + \frac{h}{2} \leq y \leq H + h$, $\frac{\partial u}{\partial y} \leq 0$ and the equation governing the velocity is as follows

$$\frac{\partial}{\partial y} \left(-\frac{\partial u}{\partial y} \right)^n = \frac{1}{m} \frac{dp}{dx} \qquad (7\ 4)$$

Integrating equation (7 4) and applying the following conditions

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = H + h/2$$

$$u = 0 \quad \text{at} \quad y = H + h,$$

we get

$$u = \left(-\frac{1}{m} \frac{dp}{dx} \right)^{1/n} \frac{\left[\left(\frac{h}{2} \right)^{1+\frac{1}{n}} - \left\{ y - \left(H + \frac{h}{2} \right) \right\}^{1+\frac{1}{n}} \right]}{1+\frac{1}{n}} \qquad (7\ 5)$$

Now, on integrating the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (7\ 6)$$

we get,

$$(-v)\Big|_H^{H+h} = \frac{\partial}{\partial x} \left\{ \int_H^{H+h/2} u\,dy + \int_{H+h/2}^{H+h} u\,dy \right\}$$

which on using equations (7 3) and (7 5) gives

$$\left(\frac{h}{2}\right)^{2+\frac{1}{n}} \frac{n}{2n+1} \frac{d}{dx}\left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n} = v_a', \tag{7 7}$$

where v_a' is the average velocity with which the lubricant is injected into the film

7 2 MODIFIED FORM OF DARCY'S LAW

Let us consider that the porous matrix consists of a system of capillaries whose axes are directed towards the film The motion of the lubricant in a typical capillary in the lower plate is governed by [see figure no (7,2)]

$$-\frac{dp'}{dy} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau'_{ry}) = 0 , \tag{7 8}$$

where $\tau'_{ry} = m\left(\frac{\partial v'}{\partial y}\right)^n$, since $\frac{\partial v'}{\partial r} \leq 0$

Integrating equation (7 8) and using the boundary conditions

$$\begin{aligned} \frac{\partial v'}{\partial r} &= 0 \quad , \; r = 0 \\ v' &= 0 \quad , \; r = R \end{aligned} \tag{7 9}$$

we get

$$v' = \left(-\frac{1}{2m}\frac{dp'}{dy}\right)^{\frac{1}{n}} \frac{R^{1+\frac{1}{n}} - r^{1+\frac{1}{n}}}{1+\frac{1}{n}} \tag{7 10}$$

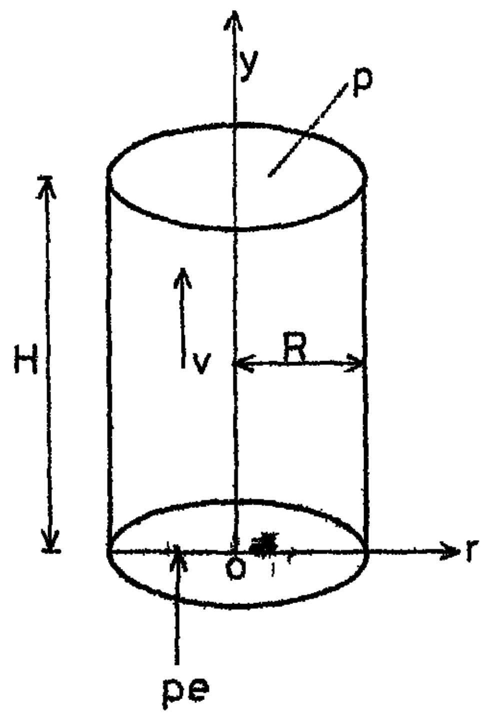


FIG 7 2 FLOW OF A POWER LAW LUBRICANT IN A THIN POROUS CAPILLARY

The flux may be defined as

$$Q' = \int_0^R 2\pi\, rv'\, dr ,$$

which, on using equation (7 10), gives

$$\frac{dp'}{dy} = -2m \left\{ \frac{(3n+1)\; Q'}{n\pi \quad R^{3+\frac{1}{n}}} \right\}^n \tag{7 11}$$

where Q' is a constant as can be seen from the equation of continuity

Integrating equation (7 11) and using the conditions $p' = p_e$ at $y = 0$ and $p' = p$ at $y = H$, we get

$$Q' = \frac{n\pi}{3n+1} \left(\frac{p_e - p}{2mH} \right)^{\frac{1}{n}} R^{3+\frac{1}{n}} \tag{7 12}$$

The average velocity v_a' of the lubricant can then be given by

$$v_a' = \frac{Q'}{\pi R^2} = \frac{n\, R_a^{1+\frac{1}{n}}}{3n + 1} \left(\frac{p_e - p}{2mH} \right)^{\frac{1}{n}} , \tag{7 13}$$

where R_a is the average radius of the system of capillaries This equation may be interpreted as a modified form of Darcy's law for power law fluids

7 3 DETERMINATION OF LOAD CAPACITY

From equations (7 7) and (7 13) we obtain the basic equation governing the pressure in the film as follows:

$$\frac{d}{dx}\left(-\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}} = \frac{2n+1}{3n+1}\left(\frac{2}{h}\right)^{2+\frac{1}{n}} R_a^{1+\frac{1}{n}} \left(\frac{p_e - p}{2mH}\right)^{\frac{1}{n}} \tag{7 14}$$

Due to symmetry, the boundary conditions for the pressure are

$$\begin{aligned} &\frac{dp}{dx} = 0, \ p = p_m \ , \ x = 0 \\ &p \ = 0 \ , \qquad\qquad x = \frac{L}{2} \end{aligned} \tag{7 15}$$

where p_m is the maximum pressure

Introducing the variable $\xi = \frac{2x}{L}$ and $P = p_e - p$ in equation (7 14) we get

$$\frac{d}{d\xi}\left(\frac{dP}{d\xi}\right)^{\frac{1}{n}} = K_2 \, P^{\frac{1}{n}} \tag{7 16}$$

$$\text{where} \quad K_2 = \frac{4H}{h}\left(\frac{2n+1}{3n+1}\right)\left(\frac{R_a L}{2Hh}\right)^{1+\frac{1}{n}} \tag{7 17}$$

is a dimensionless parameter depending upon the characteristics of the porous matrix and p_e is the externally applied pressure Integrating equation (7 16) and using the conditions

$$\frac{dP}{d\xi} = 0 \quad \text{and} \quad P = P_m \quad \text{at} \quad \xi = 0$$

we get

$$\frac{dP}{d\xi} = (nK_2)^{\frac{n}{n+1}} \left(P^{1+\frac{1}{n}} - P_m^{1+\frac{1}{n}}\right)^{\frac{n}{n+1}} \tag{7 18}$$

and $$\int_{P_m}^{P} \frac{dP}{\left(P^{1+\frac{1}{n}} - P_m^{1+\frac{1}{n}}\right)^{\frac{n}{n+1}}} = (n K_2)^{\frac{n}{n+1}} \xi \tag{7 19}$$

Here $P_m = p_e - p_m$ and can be determined by the following equation which is obtained by using the boundary condition $P = p_e$ at $\xi = 1$ in equation (7 19)

$$\int_{P_m}^{p_e} \frac{dP}{\left(P^{1+\frac{1}{n}} - P_m^{1+\frac{1}{n}}\right)^{\frac{n}{n+1}}} = (nK_2)^{\frac{n}{n+1}} \tag{7 20}$$

The load capacity W is given by

$$W = 2b \int_0^{L/2} p\, dx = b\ L \int_0^1 p\ d\xi = bL \int_0^1 (p_e \quad P)\ d\xi\ ,$$

which on using equation (7 18) gives

$$\bar{W} = 1 - \frac{1}{z_0\ \alpha'\beta'} \int_1^{z_0} \frac{z\ dz}{\left(z^{1+\frac{1}{n}} - 1\right)^{\frac{n}{n+1}}}\ , \tag{7 21}$$

where $\bar{W} = \frac{W}{bLp_e}$,

$$z = \frac{P}{P_m}\ ,\quad z_o = \frac{p_e}{P_m}$$

$$\alpha' = \left[\frac{4H}{h}\ \frac{2n^2+n}{3n+1}\right]^{\frac{n}{n+1}},\ \beta' = \frac{R_a L}{2Hh} \text{ and } \alpha'\beta' = (nK_2)^{\frac{n}{n+1}}$$

Finally, the equation (7 19) for determining P_m with the above transformation could be written as follows

$$\int_1^{z_o} \frac{dz}{\left(z^{1+\frac{1}{n}}-1\right)^{\frac{n}{n+1}}} = \alpha'\beta' \tag{7 22}$$

7 4 DISCUSSION AND RESULTS

To determine the load capacity of the bearing, first the values of z_o for different sets of values of α' and β' are numerically computed from equation (7 22) and these values are then substituted to determine $\bar{W}$ numerically from equation (7 21) The variations of $\bar{W}$ with β' for different values of $\frac{H}{h}$ and n are shown in figures (7 3) and (7 4) From figures no (7 3), (7 4) and (7 5) it can be seen that the load capacity increases as n increases for fixed β' and $\frac{H}{h}$ Further, from these figures, it is clear that for fixed n, the load capacity increases as β' and $\frac{H}{h}$ increase i e as the characteristics of the porous matrix increase These results can be seen from the following approximate calculations also

Let us assume that the externally applied pressure is large i e $p_e >> p$ With this assumption, equation (7 14) becomes

$$\frac{d}{dx}\left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n} = \frac{2n+1}{3n+1}\left(\frac{2}{h}\right)^{2+\frac{1}{n}} R_a^{1+\frac{1}{n}}\left(\frac{p_e}{2mH}\right)^{\frac{1}{n}} \tag{7 23}$$

Following the earlier procedure, the load capacity under this

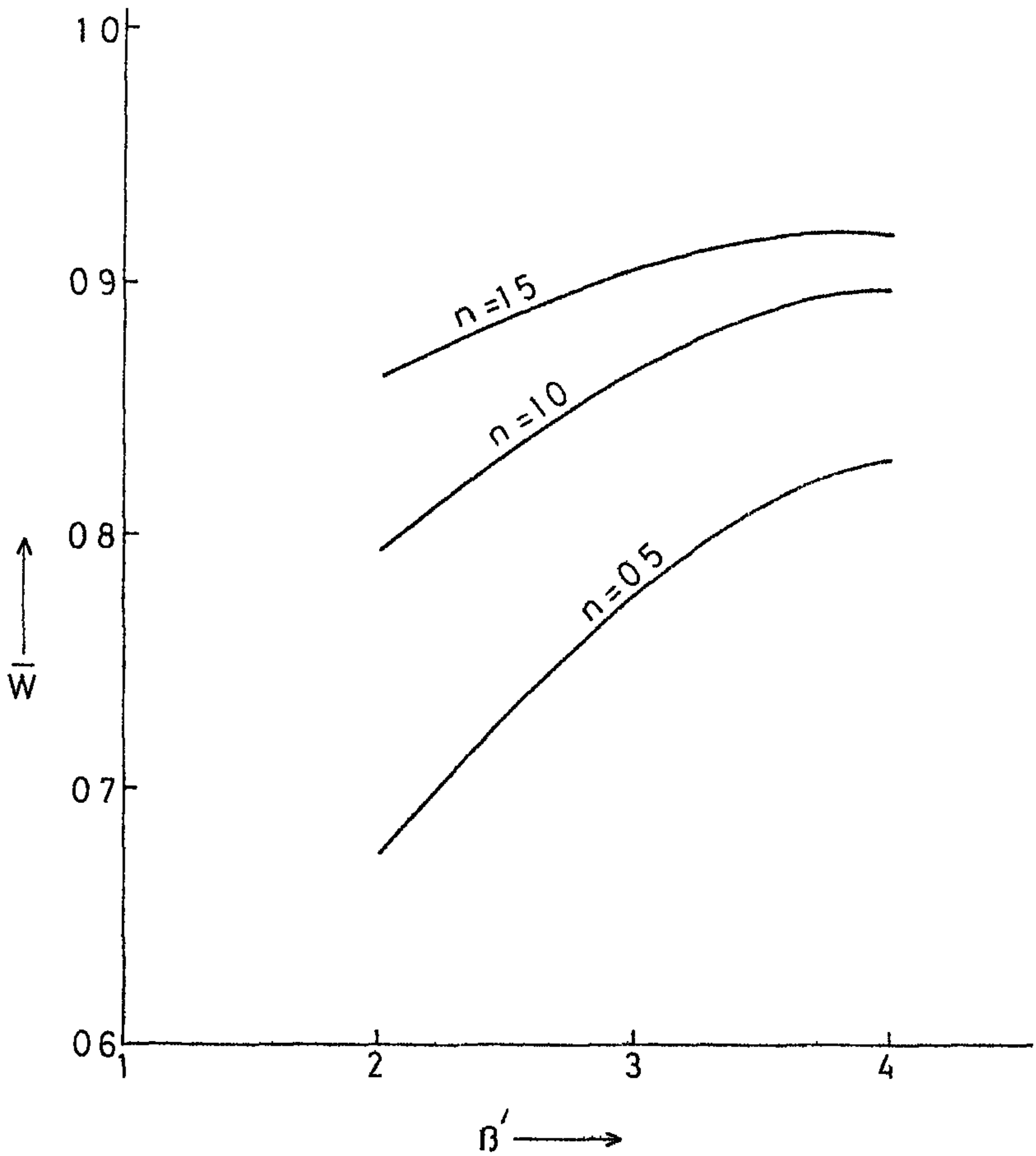


FIG 7 3 VARIATION OF $\overline{W}$ WITH β' FOR DIFFERENT n AND $\frac{H}{h}$ = 2 0

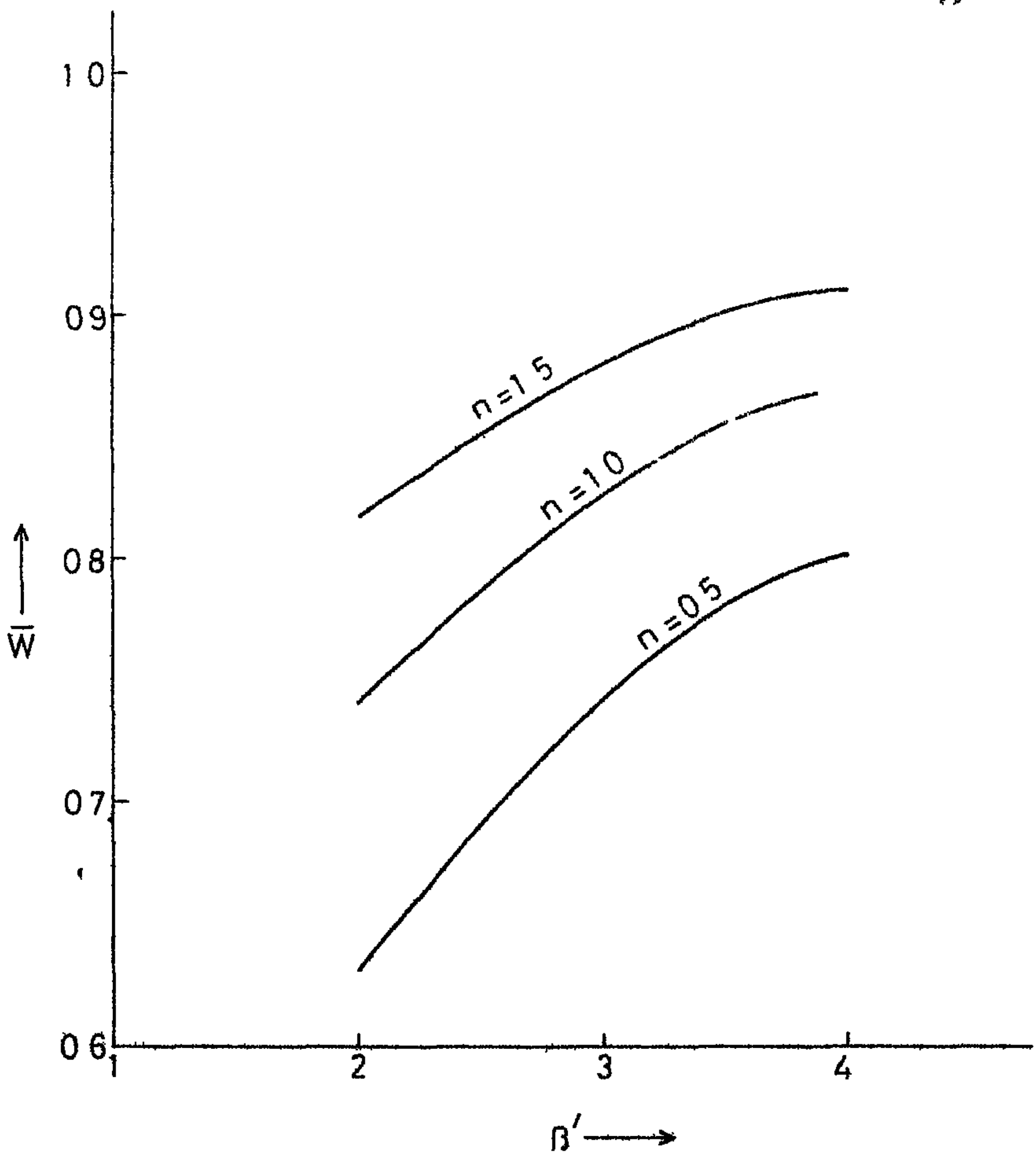


FIG 7 4 VARIATION OF $\overline{W}$ WITH ß′ FOR DIFFERENT n AND $\frac{H}{h}$ = 1 25

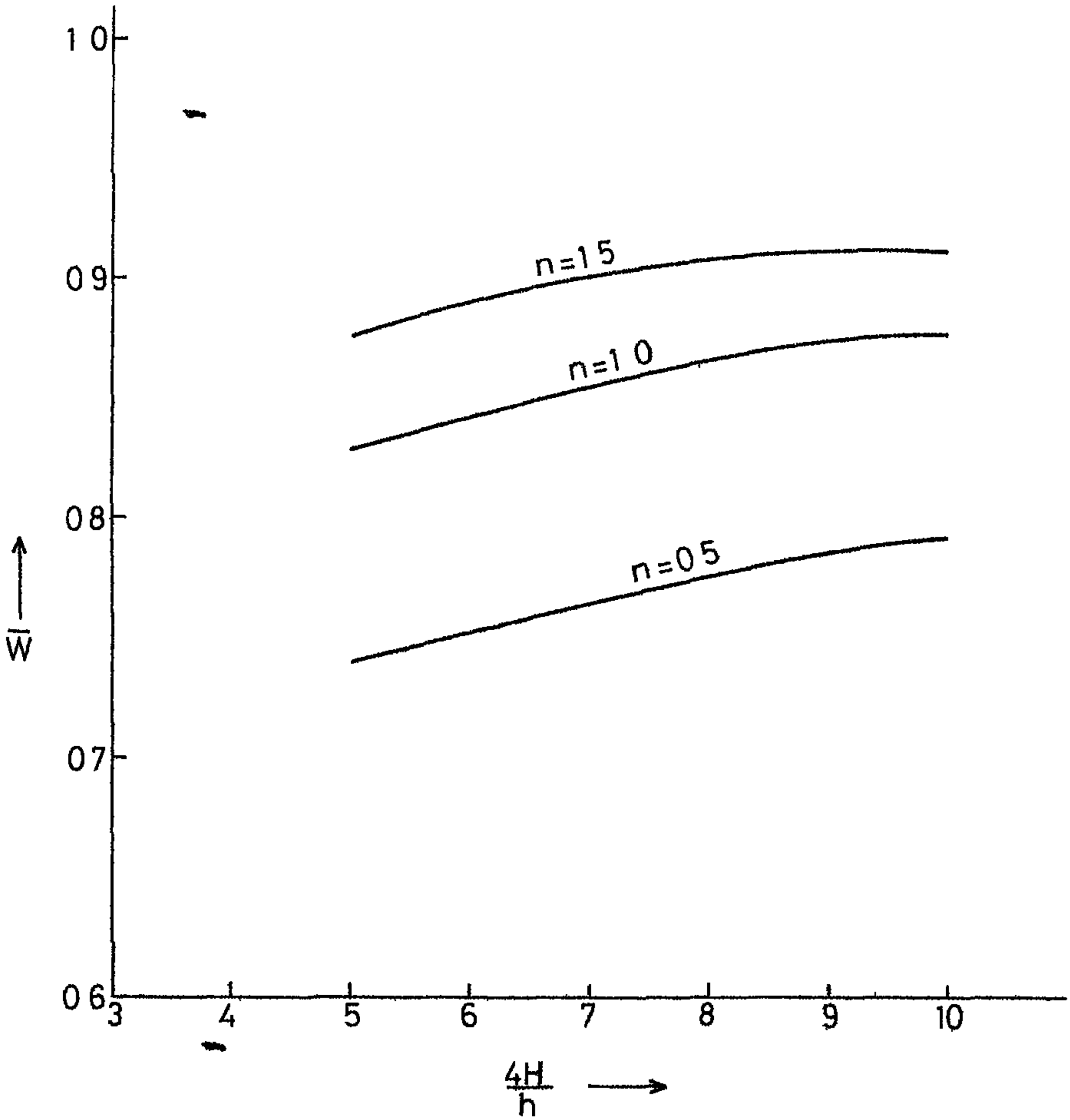


FIG 75 VARIATION OF $\bar{W}$ WITH $\frac{H}{h}$ FOR DIFFERENT n AND $\beta' = 30$

approximation can be written as follows

$$\bar{W} = \beta'^{n+1} \left(\frac{4H}{h} \right)^n \left(\frac{2n+1}{3n+1} \right)^n \frac{1}{n+2}$$

It is obvious from here that $\bar{W}$ increases as β' or $\frac{4H}{h}$ increases Further, it can be seen that $\bar{W}$ also increases as n increases for β and $\frac{4H}{h}$ greater than unity

When $n = 1$, i e the case when the lubricant is Newtonian, analytical expression for $\bar{W}$ is obtained as

$$W = 1 - \frac{\tanh \beta' \alpha_2}{\beta' \alpha_2}$$

where $$\alpha_2 = \left(\frac{3H}{h}\right)^{1/2} ,$$

which is the same as obtained by Hsing [1971] when inertia effects are neglected in his case

CHAPTER - VIII

SQUEEZE FILMS USING POWER LAW LUBRICANTS

In the previous three chapters we have studied the behaviour of power law lubricants in externally pressurised bearings and the effects of step in the film thickness, variation of consistency index with respect to temperature, and porosity of the bearing surfaces have been investigated

In this chapter, we study the characteristics of squ eze film bearings using power law lubricants by taking intertia effects and the linear variation of consistency with pressure into account The effects of step in the geometrical film thickness are also studied

In recent years, several investigators have studied the e fects of inertia in various bearings by considering Newtonian fluid as lubricant, Osterle and Saibel [1955] , Dowson [1961] , Milne [1963] , Snyder [1963] , Kuzma [1967] , Ting and Mayer [1971] The effects of viscosity variation with temperature and pressure have also been studied in the case of squeeze film bearings Gould [1967] Though the characteristics of squeeze film and other bearings using power law lubricants have been studied Ng and Saible [1962] , Shukla [1964b] , Shukla and Prakash [1969] , effects of inertia have not been taken into consideration In view of this, we study the following effects in the squeeze film bearings:

(1) Effects of inertia and pressure

(2) Effects of step in the film thickness.

The cases of infinitely extended rectangular and circular parallel plates are considered

8 1 SQUEEZE FILMS BETWEEN TWO INFINITE PLATES

Let us consider the flow of a power law lubricant between two infinitely long rectangular plates, the physical configuration of which is shown in figure no (8 1) We assume that the two plates are approaching each other with relative velocity V and the lubricant is being squeezed out due to this relative motion Making use of the symmetry of the physical situation, the basic equations governing the flow of the lubricant can be written as

$$\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = - \frac{dp}{dx} + \frac{\partial}{\partial y} \left\{-m\left(- \frac{\partial u}{\partial y}\right)^n\right\} \qquad (8\ 1)$$

$$, \quad \frac{\partial u}{\partial y} \leq 0 \text{ in } 0 \leq y \leq h/2$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (8\ 2)$$

Here the consistency index m is assumed to vary with pressure linearly as follows

$$m = m_o\ (1 + \alpha p) \qquad (8\ 3)$$

where m_o and α are constants This relation is compatible with

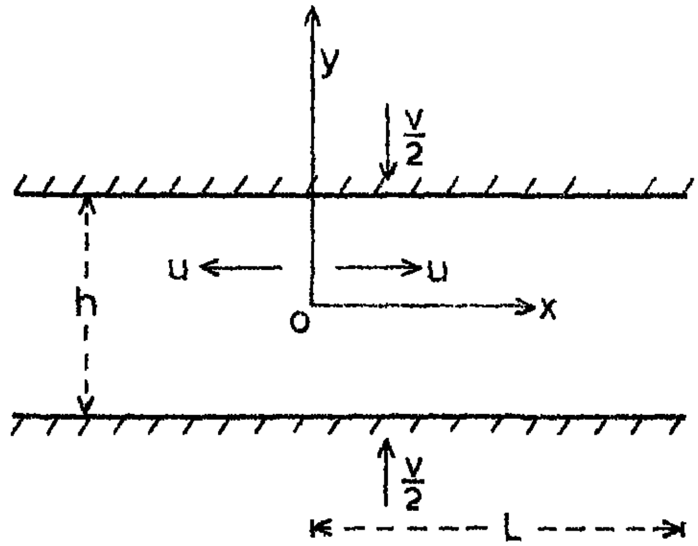


FIG 8 1 SQUEEZE FILM BETWEEN TWO INFINITE PLATES

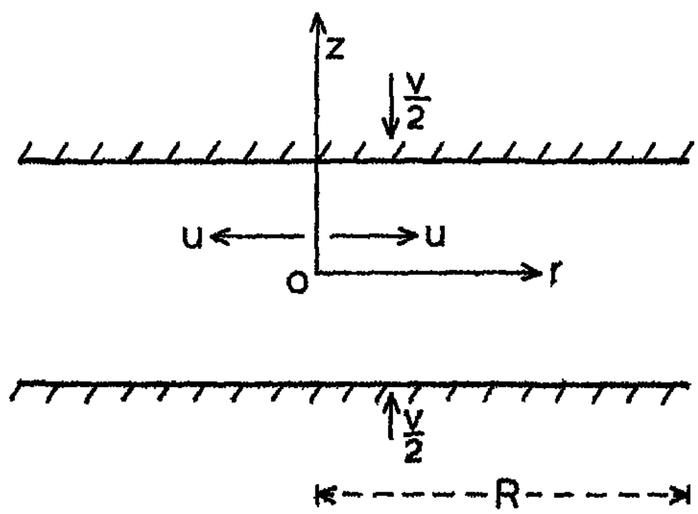


FIG 8 2 SQUEEZE FILM BETWEEN TWO CIRCULAR PLATES

the exponential variation for small values of α in which we are interested. Since the fluid film is very thin, on taking the average of inertia terms equation (8.1) can be approximated as follows Pinkus [1960]

$$\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n = -\frac{1}{m_0(1+\alpha p)}\frac{dp}{dx} \quad \frac{2\rho}{h\, m_0(1+\alpha p)}\int_0^{h/2}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)dy \qquad (8.4)$$

As p is a function of x, the right hand side of equation (8.3) can be written as a function of x and thus we have

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n = f(x) \quad , \qquad (8.5)$$

where $f(x) = -\dfrac{1}{m_0(1+\alpha p)}\dfrac{dp}{dx} \quad \dfrac{2\rho}{h\, m_0(1+\alpha p)}\displaystyle\int_0^{h/2}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)dy$

The boundary conditions for u are

$$\begin{aligned} u &= 0 \qquad \text{at} \quad y = h/2 \, , \\ \frac{\partial u}{\partial y} &= 0 \qquad \text{at} \quad y = 0 \end{aligned} \qquad (8.6)$$

Solving equation (8.5) and using the conditions (8.6) we obtain the expression for u as

$$u = f^{\frac{1}{n}} \; \frac{\left(\frac{h}{2}\right)^{1+\frac{1}{n}} - y^{1+\frac{1}{n}}}{1+\frac{1}{n}} \qquad (8.7)$$

Substituting for $\frac{\partial u}{\partial x}$ from equation (8 7) in equation (8 2) and using the boundary conditions

$$v = 0 \quad \text{at} \quad y = 0 ,$$
$$v = -\frac{V}{2} \quad \text{at} \quad y = \frac{h}{2} , \qquad (8\ 8)$$

we get

$$v = -\frac{f' f^{\frac{1}{n}-1}}{n+1} \{ y\left(\frac{h}{2}\right)^{1+\frac{1}{n}} - \frac{y^{2+\frac{1}{n}}}{2+\frac{1}{n}} \} \qquad (8\ 9)$$

and $$f' f^{\frac{1}{n}-1} = \frac{(2n+1)V}{2} \left(\frac{2}{h}\right)^{2+\frac{1}{n}} \qquad (8\ 10)$$

Integration of equation (8 10) gives

$$f = \{\frac{(2n+1)V}{2n}\}^{n} \left(\frac{2}{h}\right)^{2n+1} (x+c)^{n} \qquad (8\ 11)$$

when c is a constant to be determined

From equations (8 2), (8 5), (8 7) and (8 11) we get

$$-\frac{dp}{dx} = m_0 (1+ \alpha p) \left(\frac{2n+1}{2n} V\right)^{n} (x+c)^{n} \left(\frac{2}{h}\right)^{2n+1}$$
$$+ 4\rho \frac{2n+1}{3n+2} (x+c) \frac{V^2}{h^2} \qquad (8\ 12)$$

The boundary conditions for p are

$$p = 0 \quad \text{at} \quad x = L' ,$$
$$\frac{dp}{dx} = 0 \quad \text{at} \quad x = 0 \qquad (8\ 13)$$

Using the second condition of (8 13) and keeping in view the case $n = 1$, we get $c = 0$ Then equation (8 12) gives,

$$-\frac{dp}{dx} = m_o(1+\alpha p)\left(\frac{2n+1}{2n}\, V\right)^n \left(\frac{2}{h}\right)^{2n+1} x^n + \frac{4\rho(2n+1)}{3n+2}\,\frac{V^2}{h^2}\, x \qquad (8\ 14)$$

Integrating equation (8 14),and now using the first condition of (8 13), we have the final expression for p as

$$p = \frac{a_1}{n+1}\{L^{n+1} - x^{n+1}\} + \frac{a_2}{2}\{L^2 - x^2\} + \frac{a_1^2\alpha}{2(n+1)^2}\{L^{n+1} - x^{n+1}\}^2$$
$$+ \frac{a_1 a_2 \alpha}{2(n+1)(n+3)}\{(n+3)L^2(L^{n+1} - x^{n+1}) - (n+1)(L^{n+3} - x^{n+3})\} \qquad (8\ 15)$$

where α^2 and its higher powers have been assumed negligible in obtaining the above solution, and

$$a_1 = m_o \left(\frac{2n+1}{2n}\, V\right)^n \left(\frac{2}{h}\right)^{2n+1} ,$$
$$a_2 = 4\rho \left(\frac{2n+1}{3n+2}\right) \frac{V^2}{h^2} \qquad (8\ 16)$$

The load capacity W for width b is defined by

$$W = 2b \int_0^L p\,dx \qquad (8\ 17)$$

which on using equation (8 15) gives

$$W = 2b\left[\frac{a_1 L^{n+2}}{n+2} + \frac{a_2 L^2}{3} + \frac{a_1^2\,\alpha L^{2n+3}}{(n+2)\,(n+3)} + \frac{a_1 a_2\,\alpha L^{n+4}}{(n+2)\,(n+4)}\right] \tag{8 18}$$

The corresponding case with no pressure variation can be derived by putting $\alpha = 0$ in equation (8 18) as follows

$$W = \frac{4m_o b}{n+2}\left\{\frac{2(2n+1)V}{n}\right\}^n \left(\frac{1}{h}\right)^{2n+1} L^{n+2} + \frac{8\rho b L^3}{3}\left(\frac{2n+1}{3n+2}\right)\frac{V^2}{h^2} \tag{8 19}$$

Since this equation is too complicated to solve for V we determine the time for squeezing by neglecting inertia but taking pressure variation into account Then equation (8 18) can be simplified to

$$W = K_1 \alpha B_1^2 + K_2 B_1 , \tag{8 20}$$

where $$B_1 = \frac{a_1}{n+1}\ , \quad K_1 = \frac{2b(n+1)^2 L^{2n+3}}{(n+2)\,(2n+3)}$$

(8 21)

and $$K_2 = \frac{2b(n+1)L^{n+2}}{n+2}$$

Solving the quadratic equation (8 20) for B_1 and retaining terms of the order of α , we get

$$B_1 = \frac{W}{K_2}\left(1 - \frac{K_1 W}{K_2^2}\alpha\right) \tag{8 22}$$

Now putting $V = -\frac{dh}{dt}$ and integrating equation (8 22) we get the expression for the elapsed time required for the film thickness

to be reduced from h_1 to h_2, for a given load W, as

$$t = \frac{2(2n+1)}{n+1}\left\{\frac{4m_o b}{W(n+2)}\right\}^{\frac{1}{n}} L^{1+\frac{2}{n}}\left\{\frac{1}{h_2^{1+\frac{1}{n}}} - \frac{1}{h_1^{1+\frac{1}{n}}}\right\}\left\{1+\frac{(n+2)W\alpha}{2n(2n+3)bL}\right\} \tag{8 23}$$

When $\alpha = 0$, we get the result as obtained by Shukla [1964b]
It can be seen from equation (8 18) that the effect of inertia is to increase the load capacity of the bearing It can also be seen that the load capacity W increases as α increases When $\alpha = 0$, since $\frac{d}{dn}\left(\frac{2n+1}{3n+2}\right)$ is positive, it is obvious from equation (8 19) that the contribution to the load capacity due to inertia increases as n increases

From equation (8 23) it is easily seen that the time of squeezing also increases as α increases but the contribution due to α decreases as n increases, since $\frac{d}{dn}\left\{\frac{n+2}{n(2n+3)}\right\}$ is negative

8 2 SQUEEZE FILMS BETWEEN TWO CIRCULAR PLATES

Consider two circular flat plates of radii R approaching each other with relative velocity V The clearance space between the plates is filled with non-Newtonian power law lubricant which is being radially displaced outward by the relative motion of the plates The physical configuration is illustrated in figure no (8 2)

Taking into account the inertia effects, the basic equations governing the flow of the lubricant in the upper half of the film are

$$\frac{\partial}{\partial z}\{-m(-\frac{\partial u}{\partial z})^n\} = \frac{dp}{dr} + \rho(u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z}) \qquad (8\ 24)$$

$$,\frac{\partial u}{\partial z} \leq 0 \quad \text{in } 0 \leq z \leq h/2$$

and $$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial v}{\partial z} = 0 \qquad (8\ 25)$$

Here also we assume that the consistency index m varies linearly with pressure as given by equation (8 3) Again averaging inertia terms across the film thickness, from equations (8 3) and (8 24), we have

$$\frac{\partial}{\partial z}(-\frac{\partial u}{\partial z})^n = \phi(r), \qquad (8\ 26)$$

where $$\phi(r) = -\frac{1}{m_o(1+\alpha p)}\frac{dp}{dr} - \frac{2\rho}{m_o h(1+\alpha p)}\int_0^{h/2}(u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z})\,dz \qquad (8\ 27)$$

Solving equation (8 27) with the boundary conditions:

$$\left.\begin{aligned} u &= 0 \quad \text{at } z = h/2, \\ \frac{\partial u}{\partial z} &= 0 \quad \text{at } z = 0 \quad , \end{aligned}\right\} \qquad (8\ 28)$$

we have

$$u = \phi^{\frac{1}{n}}\ \frac{(\frac{h}{2})^{1+\frac{1}{n}} - z^{1+\frac{1}{n}}}{1+\frac{1}{n}} \qquad (8\ 29)$$

Now substituting u from equation (8 29) in equation (8 25) and integrating with following boundary conditions:

$$v = 0 \quad \text{at} \quad z = 0$$
$$v = -\frac{V}{2} \text{ at } \quad z = \frac{h}{2} \tag{8 30}$$

we have

$$v = -\frac{1}{r}\{\frac{d}{dr}(r\phi^{1/n})\}\frac{1}{1+\frac{1}{n}}\{z(\frac{h}{2})^{1+\frac{1}{n}} - \frac{z^{2+\frac{1}{n}}}{2+\frac{1}{n}}\} \tag{8 31}$$

and
$$\frac{1}{r}\ \frac{d}{dr}(r\phi^{\frac{1}{n}}) = \frac{(2n+1)V}{2n}\left(\frac{2}{h}\right)^{2+\frac{1}{n}} \tag{8 32}$$

Integrating again equation (8 32) we get,

$$\phi = [\frac{(2n+1)V}{4n}]^n \ [\frac{2}{h}]^{2n+1}\ (r + \frac{A}{r})^n , \tag{8 33}$$

where A is a constant be determined

From equations (8 25), (8 28) and (8 30) we can write

$$\int_0^{h/2} (u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z})\, dz = \int_0^{h/2} (2u\frac{\partial u}{\partial r} + \frac{u^2}{r})\, dz \tag{8 34}$$

which on using equations (8 26), (8 27), (8 29) and (8 34) gives

$$\frac{dp}{dr} = \eta_o(1+\alpha\omega)[\frac{(2n+1)V}{4n}]^n(\frac{2}{h})^{2n+1}(r + \frac{A}{r})^n + \frac{\rho}{2}\,\frac{2n+1}{3n+2}\,\frac{V^2}{h^2}(3r + \frac{2A}{r} - \frac{A^2}{r^3}) \tag{8 35}$$

The boundary conditions for p are

$$p = 0 \quad \text{at} \quad r = R\ ,$$
$$\frac{dp}{dr} = 0 \quad \text{at} \quad r = 0 \qquad (8\ 36)$$

The second condition of (8 36) when used in (8 35) implies that $A = 0$ and we have,

$$-\frac{dp}{dr} = m_o(1+\alpha p)\left[\frac{(2n+1)V}{4n}\right]^n \left(\frac{2}{h}\right)^{2n+1} r^n + \frac{3\rho}{2}\,\frac{2n+1}{3n+2}\,\frac{V^2}{h^2}\, r \qquad (8\ 37)$$

Integrating equation (8 37), using the first condition of equation (8 36) and retaining terms upto α , we get the final expression for p as

$$p = \frac{a_3}{n+1}\left(R^{n+1} - r^{n+1}\right) + \frac{a_4}{2}\left(R^2 - r^2\right) + \frac{a_3^2\,\alpha}{2(n+1)^2}\left\{R^{n+1} - r^{n+1}\right\}^2$$
$$+ \frac{a_3 a_4\,\alpha}{2(n+1)\,(n+3)}\left\{(n+3)R^2\left(R^{n+1} - r^{n+1}\right) - (n+1)\left(R^{n+3} - r^{n+3}\right)\right\} \qquad (8\ 38)$$

where $$a_3 = 2m_o\left[\frac{(2n+1)V}{n}\right]^n \left(\frac{1}{h}\right)^{2n+1} \qquad (8\ 39)$$

and $$a_4 = \frac{3\rho}{2}\ \frac{2n+1}{3n+1}\ \frac{V^2}{h^2}$$

The instanteneous load capacity W for a given squeeze velocity is given by

$$W = \int_0^R 2\pi r\, p \, dr \;, \qquad (8\ 40)$$

which on using equation (8 38) gives

$$W = 2\pi \left[\frac{a_3 R^{n+3}}{2(n+3)} + \frac{a_4 R^4}{8} + \frac{a_3^2\, \alpha\, R^{2n+4}}{4(n+2)(n+3)} + \frac{a_3 a_4 \alpha R^{n+5}}{2(n+3)(n+5)}\right] \qquad (8\ 41)$$

The load capacity for constant consistency index is obtained by putting $\alpha = 0$ in equation (8 41) as follows

$$W = \frac{2\pi\, m_0}{n+3} \left(\frac{2n+1}{n} V\right)^n \left(\frac{1}{h}\right)^{2n+1} R^{n+3} + \frac{3\rho\pi}{8}\, \frac{2n+1}{3n+2}\, \frac{V^2}{h^2}\, R^4 \qquad (8\ 42)$$

Equation (8 41) is the general non-linear equation for determining V and hence the squeezing time However , to see the effect of α on the squeezing time, we neglect the inertia terms in equation (8 41) and we then have,

$$K_3 \alpha B_2^2 + K_4 B_2 - W = 0 \qquad (8\ 43)$$

where $\quad B_2 = \dfrac{a_3}{n+1}$,

$$K_3 = \frac{\pi}{2} \; \frac{(n+1)^2}{(n+2)(n+3)} \; R^{2n+4} \;, \qquad (8\ 44)$$

and $\quad K_4 = \dfrac{\pi(n+1)}{n+3} R^{n+3}$

Solving equation (8 43) for B_2 and retaining terms upto α only, we get

$$B_2 = \frac{W}{K_4} \left(1 - \frac{WK_3}{K_4^2} \alpha \right) \quad (8\ 45)$$

and then as before, the squeezing time can be written as follows

$$t = \frac{2n+1}{n+1} \left[\frac{2\pi\ \eta_o}{(n+3)W}\right]^{\frac{1}{n}} p^{1+\frac{3}{n}} \left(\frac{1}{h_2^{1+\frac{1}{n}}} \quad \frac{1}{h_1^{1+\frac{1}{n}}}\right) \left[1 + \frac{W}{2\pi} \frac{n+3}{n(n+7)} \frac{\alpha}{p^2}\right] \quad (8\ 46)$$

We get the results already obtained by Shukla [1964b] when $\alpha = 0$ As in the previous section, it can be seen from equation (8 41) that the effect of inertia is to increase the load capacity It is further noticed that the load capacity increases as α increases In the absence of inertia term, the squeezing time increases as α increases and this increase decreases as n increases

8 3 EFFECTS OF STEP IN FILM THICKNESS

In this section the effects of step in the film thickness of squeeze film bearing are studied The cases of two infinite plates and two circular plates are considered:

A SQUEEZE FILM BETWEEN TWO STEPPED PARALLEL PLATES

Consider that a power law fluid is being squeezed out in a stepped clearance between two infinite plates due to normal motion of the upper plate as shown in figure no (8 3)

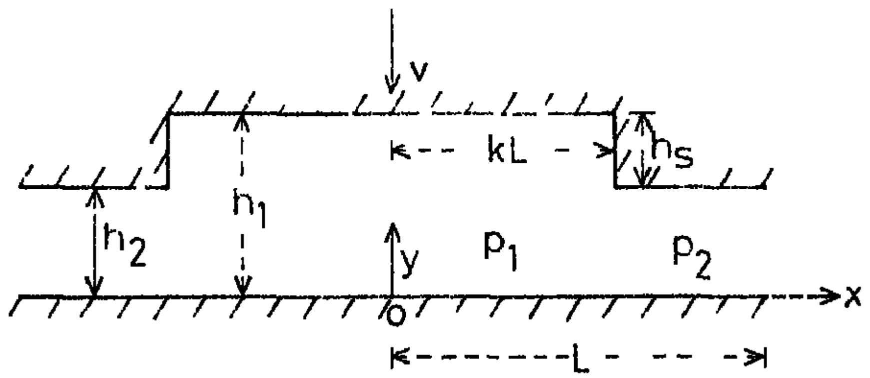


FIG 8 3 SQUEEZE FILM BETWEEN STEPPED PARALLEL PLATES

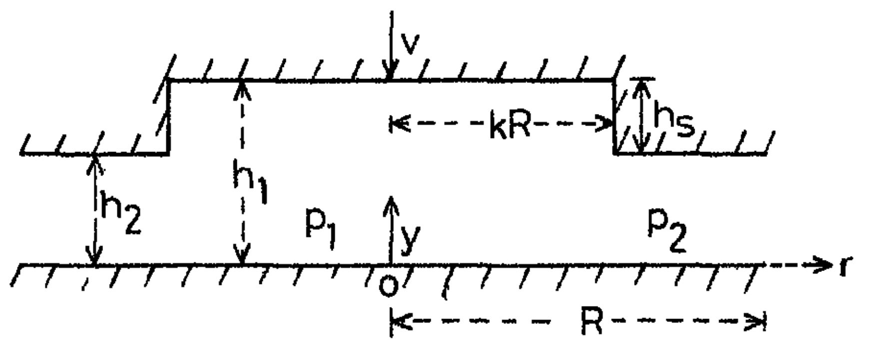


FIG 8 4 SQUEEZE FILM BETWEEN STEPPED PARALLEL CIRCULAR PLATES

In this case the fluid flow is symmetrical with respect to the line $x = 0$ and the film thickness in the two regions are given by

$$\begin{aligned} h &= h_1 \quad , \quad 0 \leq x \leq kL \\ h &= h_2 \quad , \quad kL \leq x \leq L \end{aligned} \tag{8 47}$$

If Q_1, Q_2 and p_1, p_2 are the volume fluxes and pressures respectively in the two regions, then from equation (5 15) we have,

$$\frac{\partial Q_j}{\partial x} = bV \tag{8 48}$$

where $j = 1$ for $0 \leq x \leq kL$

and $j = 2$ for $kL \leq x \leq L$

Integrating (8 48) we get

$$Q_j = bVx + A_j \tag{8 49}$$

Since $Q_1 = Q_2$ at $x = kL$, we get

$$\begin{aligned} Q_1 &= b\ Vx + A\ , \quad 0 \leq x \leq kL \\ Q_2 &= b\ Vx + A \quad kL \leq x \leq L\ , \end{aligned} \tag{8 50}$$

where $A_1 = A_2 = A$ is a constant This constant is zero as $Q_1 = 0$ at $x = 0$

Thus, finally Q_j can be written as

$$Q_j = b\ Vx \tag{8 51}$$

From equations (5 13) and (8 51), the equation determining the pressure is given by

$$\frac{dp_j}{dx} = - m_o \left(\frac{2n+1}{2n} V \right)^n \left(\frac{2}{h_j} \right)^{2n+1} x^n \quad , \; j = 1,2 \tag{8 52}$$

Integrating this and using the boundary conditions

$$p_1 = p_2 \quad \text{at} \quad x = kL$$

$$p_2 = 0 \quad \text{at} \quad x = L$$

we get

$$p_1 = \frac{m_o}{n+1} \left(\frac{2n+1}{2n} V \right)^n \left[\left(\frac{2}{h_1} \right)^{2n+1} \{ (kL)^{n+1} - x^{n+1} \} + \left(\frac{2}{h_2} \right)^{2n+1} L^{n+1} (1 - k^{n+1}) \right] \tag{8 53}$$

$$p_2 = \frac{m_o}{n+1} \left(\frac{2n+1}{2n} V \right)^n \left(\frac{2}{h_2} \right)^{2n+1} (L^{n+1} - x^{n+1}) \tag{8 54}$$

The load capacity W for width b is given by

$$W = 2b \int_0^{kL} p_1 dx + 2b \int_{kL}^{L} p_2 dx,$$

which on using equations (8 53) and (8 54) gives

$$W = \frac{2bm_o}{n+2} \left(\frac{2n+1}{2n} V \right)^n L^{n+2} F \tag{8 55}$$

where

$$F = \left(\frac{2}{h_2 + h_s} \right)^{2n+1} k^{n+2} + \left(\frac{2}{h_2} \right)^{2n+1} (1 - k^{n+2}) \tag{8 56}$$

and $\qquad h_1 = h_2 + h_s$

The squeezing time t for reducing the film thickness from an initial value h_i to a final value h_f for a steady load W is given by writing $V = - \frac{dh_2}{dt}$ as follows

$$t = \frac{2n+1}{2n} \left[\frac{2bm_o}{(n+2)W}\right]^{\frac{1}{n}} L^{1+\frac{2}{n}} \int_{h_f}^{h_i} F^{\frac{1}{n}}\, dh_2 \qquad (8\ 57)$$

From equation (8 56) it can be very easily seen that both $\frac{\partial F}{\partial h_s}$ and $\frac{\partial F}{\partial k}$ are negative for given h_2, m and n This implies that F decreases as h_s or k increases

Thus, from equations (8 55) and (8 57) it can be concluded that both the load capacity W and the time of approach t decrease with the increase either in h_s in k This result is also obvious from the following approximate expression for t when $\frac{h_s}{h_f} << 1$:

$$t = \frac{2(2n+1)}{n} \left[\frac{4bm_o}{(n+2)W}\right]^{\frac{1}{n}} L^{1+\frac{2}{n}} \left[\frac{n}{n+1} \left\{\frac{1}{h_f^{1+\frac{1}{n}}} - \frac{1}{h_i^{1+\frac{1}{n}}}\right\} - h_s k^{n+2} \left\{\frac{}{h_f^{2+\frac{1}{n}}} - \frac{1}{h_i^{1+\frac{1}{n}}}\right\}\right] \qquad (8\ 58)$$

The other characteristics of W and t are similar to those obtained by Shukla [1964b]

B SQUEEZE FILM BETWEEN TWO STEPPED PARALLEL CIRCULAR PLATES

The squeezing flow of a power law fluid between two stepped parallel circular plates is shown in figure no (8 4)

The equation governing the flux in this case can be written (from equation (5 13) and putting $b = 2\pi r$) as follows

$$Q_j = \frac{4n\pi}{2n+1} \left(-\frac{1}{m_o} \frac{dp_j}{dr}\right)^{\frac{1}{n}} \left(\frac{h_j}{2}\right)^{2+\frac{1}{n}}, \quad j = 1\ 2 \tag{8 59}$$

where p_1, Q_1 and p_2, Q_2 are the pressures and the fluxes in the regions $h = h_1$ $0 \leq x \leq kR$ and $h = h_2$, $kR \leq r \leq R$ respectively Further, Q_j can also be obtained from equation (5 15) as

$$\frac{\partial Q_j}{\partial r} = 2\pi\ rV \tag{8 60}$$

As in the previous case, since the flux Q_1 is zero at $r = 0$ and $Q_1 = Q_2$ at the step $r = kR$, we have on integrating equation (8 60)

$$Q_j = \pi\ r^2 V \tag{8 61}$$

Then, from equations (8 59) and (8,61) we get the equation determining the pressure as follows:

$$\frac{dp_j}{dr} = -\ m_o \left(\frac{2n+1}{4n}\ V\right)^n \left(\frac{2}{h_j}\right)^{2n+1} r^n, \quad j = 1,2 \tag{8 62}$$

Integrating this and using the boundary conditions

$$\begin{aligned} p_1 &= p_2 \quad \text{at} \quad r = kR \\ p_2 &= 0 \quad \text{at} \quad r = R \end{aligned} \tag{8 63}$$

we have

$$p_1 = \frac{m_o}{n+1} \left(\frac{2n+1}{4n}\ V\right)^n \left[\left(\frac{2}{h_1}\right)^{2n+1} \{(kR)^{n+1} - r^{n+1}\} + \left(\frac{2}{h_2}\right)^{2n+1} R^{n+1} (1-k^{n+1})\right] \tag{8 64}$$

$$p_2 = \frac{m_o}{n+1} \left(\frac{2n+1}{4n} V\right)^n \left[\left(\frac{2}{h_2}\right)^{2n+1} \{R^{n+1} - r^{n+1}\}\right] \tag{8.65}$$

The load capacity W is defined by

$$W = \int_0^{kR} p_1 \, 2\pi \, rdr + \int_{kR}^{R} p_2 2\pi rdr \,, \tag{8.66}$$

which on using equations (8.64) and (8.65), gives

$$W = \frac{\pi m_o}{n+3} \left(\frac{2n+1}{4n} V\right)^n R^{n+3} G \,, \tag{8.67}$$

where

$$G = \left(\frac{2}{h_2 + h_s}\right)^{2n+1} k^{n+3} + \left(\frac{2}{h_2} \right)^{2n+1} (1-k^{n+3}) \tag{8.68}$$

As in the case of parallel plates, the time of approach is given by

$$t = \frac{2n+1}{4n} \left[\frac{\pi m_o}{(n+3)W}\right]^{\frac{1}{n}} R^{1+\frac{3}{n}} \int_{h_f}^{h_1} G^{\frac{1}{n}} \, dh_2 \tag{8.69}$$

Here the behaviour of G with respect to h_s or k is the same as that of F in the previous case. Hence in this case also both the load capacity and the time of approach decrease as h_s or k increases. This result for time of approach can also be concluded from the following approximate expression for t when $\frac{h_s}{h_f} << 1$:

$$t = \frac{2n+1}{n}\left[\frac{2\pi m_o}{(n+3)W}\right]^{\frac{1}{n}} R^{1+\frac{3}{n}}\left[\frac{n}{n+1}\left\{\frac{1}{h_f^{1+\frac{1}{n}}} - \frac{1}{h_i^{1+\frac{1}{n}}}\right\} - h_s\, k^{n+3}\left\{\frac{1}{h_f^{2+\frac{1}{n}}} - \frac{1}{h_i^{2+\frac{1}{n}}}\right\}\right] \qquad (8\ 70)$$

When $h_s = 0$ in both the above cases the results are the same as obtained by Shukla [1964b]

8 4 CONCLUSIONS

In this chapter the effects of inertia and pressure on the characteristics of squeeze film bearings with constant film thickness using power law lubricants are studied It is found that the effect of inertia is to increase the load capacity of the squeeze film bearing in both the cases On considering the linear variation of consistency index with the pressure i e $m = m_o(1 + \alpha p)$, it has further been pointed out that the load capacity increases as the coefficient α of pressure increases In the case, when inertia is neglected, it is seen that the squeezing time increases as α increases

It has also been shown that in the absence of inertia, and consistency variation with pressure, both the load capacity and time of approach decrease as the step height increases or as the step location moves away from the central position

CHAPTER - IX

OPTIMUM EXTERNALLY PRESSURISED MAGNETOHYDRODYNAMIC BEARING

As described in Chapter I, the problem of hydromagnetically lubricated externally pressurised bearing with constant film thickness has been studied by Hughes and Elco [1962], Shukla and Prasad [1966] and others, Kamiyama [1969-70-71] In this chapter, we consider the case of an externally pressurised bearing with variable film thickness in the presence of a non-uniform magnetic field applied in the axial direction The bearing surfaces are assumed to be non-conducting and no external current is applied Due to external pressurisation, the lubricant flows symmetrically in the radial direction and since the magnetic field is applied in the axial direction, a body force is generated in the film modifying the pressure of the lubricant The physical situation is illustrated in figure no (9 1) The calculus of variation technique, as used by Rayleigh [1918], Osterle and Young [1962] and Walker and Osterle [1961] has been applied to determine the optimum profiles for film thickness and magnetic field for maximum load capacity of the bearing under a given supply pressure

9 1 BASIC EQUATIONS:

The basic equation governing the flow of a conducting incompressible lubricant, under the usual assumptions of hydromagnetic lubrication: Shukla [1963], Hughes and Elco [1962], is

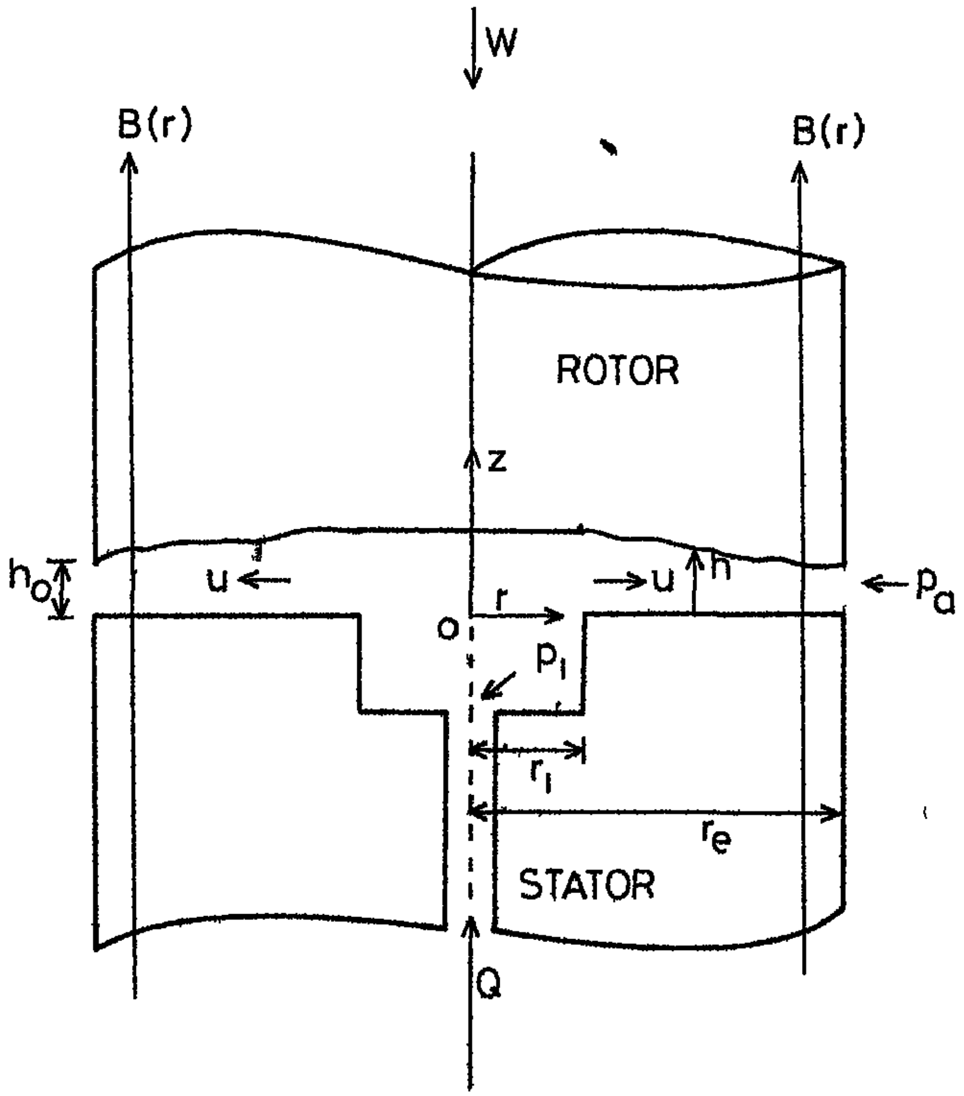


FIG 91 THE AXIAL FIELD BEARING WITH VARIABLE FILM THICKNESS

$$\frac{\partial^2 u}{\partial z^2} - \frac{M^2}{h_o^2} u = \frac{1}{\mu}\frac{dp}{dr} \quad , \tag{9 1}$$

where $\frac{dp}{dr}$ is function of r only and $M(r) = h_o B(r) (\frac{\sigma}{\mu})^{\frac{1}{2}}$ is the Hartmann number

Integrating equation (9 1) and using the boundary conditions

$$\begin{aligned} u &= 0 \quad \text{at} \quad z = 0 \\ u &= 0 \quad \text{at} \quad z = h(r) \end{aligned} \tag{9 2}$$

we get

$$u = \frac{h_o^2}{\mu M^2}\frac{dp}{dr}\{\cosh\frac{Mz}{h_o} - \tanh\frac{Mh}{2h_o}\sinh\frac{Mz}{h_o} - 1\} \tag{9 3}$$

The flow flux Q is defined by

$$Q = \int_0^h 2\pi r \, u dz \quad , \tag{9 4}$$

which gives on using equation (9 3)

$$Q = \frac{4\pi r h_o^3}{\mu M^3}\frac{dp}{dr}(\tanh\frac{Mh}{2h_o} - \frac{Mh}{2h_o}) \tag{9 5}$$

Rearranging equation (9 5) we get

$$\frac{dp}{dr} = -\frac{\mu Q M^3}{4\pi r h_o^3}\,\frac{1}{\frac{Mh}{2h_o} - \tanh\frac{Mh}{2h_o}} \tag{9 6}$$

It can be seen from the equation of continuity that Q is a constant with respect to r. The boundary conditions for p are

$$p = p_i \quad \text{at} \quad r = r_i$$
$$p = p_a \quad \text{at} \quad r = r_e \tag{9.7}$$

Integrating equation (9.6) and using the condition (9.7) we get

$$p - p_a = (p_i - p_a)\left[1 - \frac{\displaystyle\int_1^x \frac{M^3\,dx}{x\left(\frac{MY}{2} - \tanh\frac{MY}{2}\right)}}{\displaystyle\int_1^b \frac{M^3\,dx}{x\left(\frac{MY}{2} - \tanh\frac{MY}{2}\right)}}\right] \tag{9.8}$$

and

$$p_i - p_a = \frac{\mu Q}{4\pi h_o^3}\int_1^b \frac{M^3}{x\left(\frac{MY}{2} - \tanh\frac{MY}{2}\right)}\,dx\,, \tag{9.9}$$

where $Y = \dfrac{h}{h_o} = Y(x)$, $x = \dfrac{r}{r_i}$ & $b = \dfrac{r_e}{r_i}$

Equations (9.8) and (9.9) can be written as

$$p - p_a = (p_i - p_a)\left[1 - \frac{\displaystyle\int_1^x \frac{F(M,Y)}{x}\,dx}{\displaystyle\int_1^b \frac{F(M,Y)}{x}\,dx}\right] \tag{9.10}$$

and

$$p_i - p_a = \frac{\mu Q}{4\pi h_o^3}\int_1^b \frac{F(M,Y)}{x}\,dx\,, \tag{9.11}$$

$$\text{where} \qquad F(M,Y) = \frac{M^3(x)}{\frac{M(x)Y(x)}{2} - \tanh \frac{M(x)Y(x)}{2}} \tag{9.12}$$

The equation (9.10) gives the pressure distribution of the lubricant while equation (9.11) describes the relation between the flux Q and the externally applied pressure p_i

The load capacity of the bearing is given by

$$W = \pi(p_i - p_a)\, r_i^2 + 2\pi\, r_i^2 \int_1^b x(p - p_a)\, dx\,, \tag{9.13}$$

which finally gives on using equation (9.10)

$$W = \pi(p_i - p_a)\, r_i^2 \, \frac{\int_1^b xF(M,Y)\,dx}{\int_1^b \frac{F(M,Y)}{x}\, dx} \tag{9.14}$$

The above expression gives the load capacity of the bearing for given functions $Y(x)$ and $M(x)$

The following two cases are studied

(A) $p_i - p_a$ is prescribed

(B) Q is prescribed

9.2 MAXIMUM LOAD CAPACITY WHEN EXTERNALLY APPLIED PRESSURE IS PRESCRIBED

Here we wish to determine the magnetic field and film thickness profiles which maximise the load capacity of the bearing by using

calculus of variations The following three cases are considered

I when the film thickness is uniform and the applied magnetic field varies with radius,

II when uniform magnetic field is applied and the film thickness varies with radius,

III when both magnetic field and film thickness vary with radius

CASE I In this case, the nondimensional film thickness Y is unity and hence from equation (9 14) the dimensionless load capacity can be rewritten as

$$\bar{W} = \frac{W}{\pi(p_1 \quad p_a)r_1^2} = \frac{\int_1^b xF(M \quad 1)\,dx}{\int_1^b \frac{F(M \quad 1)}{x}\,dx} = \frac{g_1}{f_1} \quad , \tag{9 15}$$

where $g_1 = \int_1^b x\,F(M \quad 1)\,dx \qquad f_1 = \int_1^b \frac{F(M \quad 1)}{x}\,dx$ (9 16)

and the function $M(x)$ is to be determined for maximum $\bar{W}$

Now any change δM in M would make a corresponding change $\delta\bar{W}$ in $\bar{W}$ This change $\delta\bar{W}$ can be written by using equation (9 15) follows

$$\delta\bar{W} = \frac{1}{f_1^2} \int_1^b G_1(x) \frac{d}{dM} F(M \quad 1)\;\delta M\,dx \tag{9 17}$$

where $G_1(x) = \frac{g_1}{x} - xf_1$ (9 18)

It can be seen from equation (9 12) that $F(M,1)$ is an increasing function of M for $M \geq 0$ is and so $\frac{d}{dM} F(M,1) \geq 0$ for $M \geq 0$ This means that the sign of $\delta \bar{W}$ is given by the product of $G_1(x)$ and δM Now the optimum magnetic field is that value of M for which $\delta \bar{W}$ is negative for any admissible δM

Following Rayleigh [1918] we see that $G_1(x)$ is positive for values of x close to unity and negative for values close to b Suppose the value of x at which $G_1(x)$ changes sign is 'a_m', then from equation (9 18)

$$a_m^2 = \frac{g_1}{f_1} \tag{9 19}$$

The condition that $\delta \bar{W}$ is negative for any admissible δM would be satisfied if we consider the following magnetic field profile:

$$\begin{aligned} M &= 0 \ , \ 1 \leq x \leq a_m \\ M &= M_o \ , \ a_m \leq x \leq b \end{aligned} \tag{9 20}$$

It can be seen that for $1 \leq x \leq a_m$, δM can only be positive and since $G_1(x)$ is positive in this range, $\delta \bar{W}$ is negative For the region $a_m \leq x \leq b$, δM can only be negative and since $G_1(x)$ is negative in this range $\delta \bar{W}$ is again negative Thus it has been shown that the optimum profile for magnetic field function should be a step function

The load capacity for this type of magnetic field profile is given by using equations (9 15) and (9 19) as follows

$$\bar{W}_m = a_m^2 \tag{9 21}$$

where a_m is found by using equations (9 19), (9 20) as

$$a_m^2 = \frac{\rho_1}{\epsilon_1} = \frac{\int_1^{a_m} xF(0\ 1)\ dx + \int_{a_m}^{b} x\ F(M_o\ 1)\ dx}{\int_1^{a_m} \frac{F(0,1)}{x}\ dx + \int_{a_m}^{b} \frac{F(M_o\ 1)}{x}\ dx}$$

or $$\frac{F(M_o\ 1)}{F(0\ 1)} = \frac{2a_m^2\ \ln a_m - (a_m^2 - 1)}{(b^2-a_m^2) - 2a_m^2\ \ln \frac{b}{a_m}} = S_2$$

where $F(M_o,1)$ and $F(0,1)$ are the values of $F(M,Y)$ when $Y = 1$, and $M = M_o$ and $M = 0$ respectively The equation (9 22) gives the relation between M_o and a_m for maximum load capacity Since $F(M_o,1)$ increases as M_o increases, the function S_2 also increases with M_o The variation of S_2 with $\frac{a_m}{b}$ is shown in figure no (9 2) It is seen that the function S_2 increases as a_m increases for fixed b Since from equation (9 21), the optimum load capacity is proportional to a_m^2 and therefore to $\frac{a_m}{b}$ for given b, it may be concluded that $\bar{W}_m$ increases as M_o increases

Numerical example The following example illustrates that the above chosen profile for M gives greater load capacity in comparison to uniform magnetic field

By taking the value of $M_o = 10$, in equation (9 22), we have

$\frac{F(M_o,1)}{F(0,1)} = 10\ 41 = S_2$ From figure (9 2) we note that this value

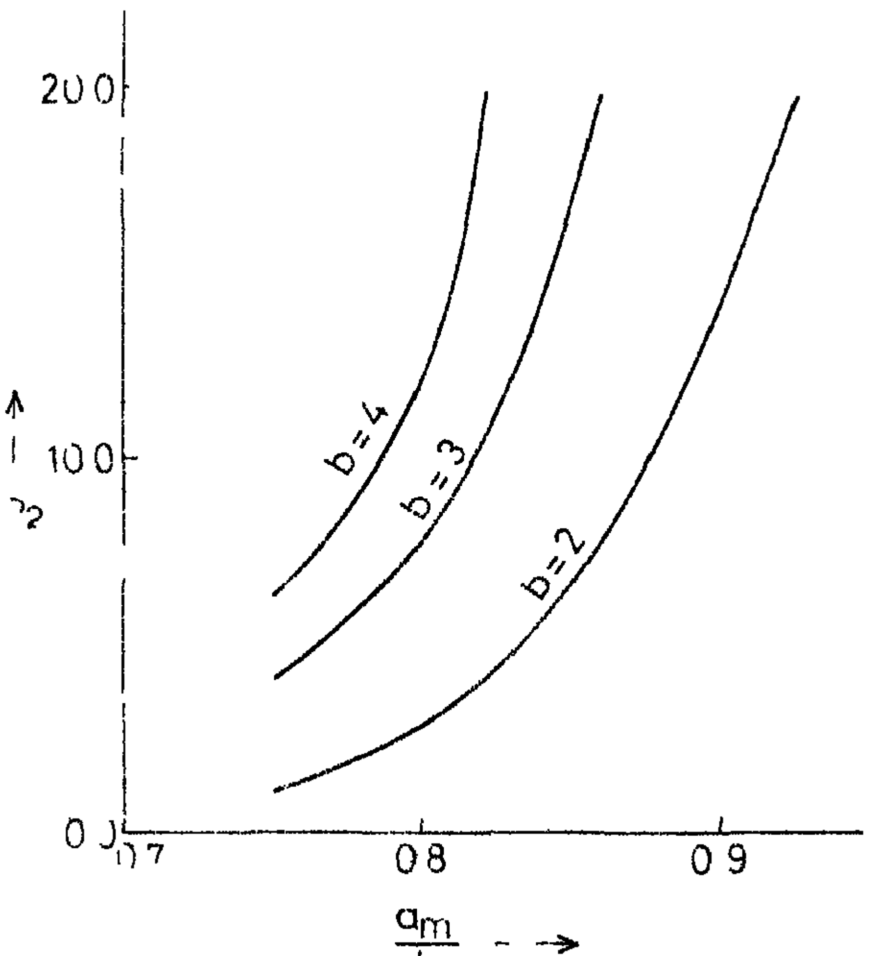


FIG 9 2 VARIATION OF S_2 WITH $\frac{a_m}{b}$ FOR DIFFERENT b

of g_2 corresponds to

$$a_m = 3.16 \text{ and then } \bar{W}_m \quad a_m^2 = 9.98$$

If we take $M = 0$ throughout (or a constant magnetic field is applied throughout), then following Hughes and Elco [1962] we have

$$\bar{W} = \frac{b^2-1}{2\ln b} = 5.41$$

Thus it is concluded that $\bar{W}_m > \bar{W}$ and the increase is about 83% for $M_o = 10$

CASE II In this case uniform magnetic field M_o is applied throughout the region $1 \leq x \leq b$, and the film thickness $Y(x)$ is a variable function of x. As before, the expression for $\bar{W}$ can be written as

$$\bar{W} = \frac{\int_1^b xF(M_o Y)\, dx}{\int_1^b \frac{F(M_o Y)}{x}\, dx} = \frac{g_2}{f_2} , \qquad (9.23)$$

where $$g_2 = \int_1^b x\, F(M_o, Y) dx$$

and $$f_2 = \int_1^b \frac{F(M_o, Y)}{x} dx \qquad (9.24)$$

As in Case I any change δY in Y will give rise to a corresponding change $\delta\bar{W}$ in $\bar{W}$. Then from equation (9.23) $\delta\bar{W}$ given by

$$\delta\bar{W} = \frac{M_o^4}{2f_2^2} \int_1^b \phi_2(x) \frac{\tanh^2(\frac{M_oY}{2})}{(\frac{M_oY}{2} - \tanh\frac{M_oY}{2})^2} \delta Y\, dx \qquad (9.25)$$

where $\quad G_2(x) = \frac{g_2}{x} - xf_2$ (9 26)

By applying similar reasoning as given in Case I, it can be seen that the following profile for film thickness gives the maximum load capacity for uniform magnetic field

$$\begin{aligned} Y &= Y_o \quad \text{in} \quad 1 \le x \le a \\ Y &= 1 \quad \text{in} \quad a \le x \le b \end{aligned} \qquad (9\ 27)$$

where Y_o is the ratio of the maximum to minimum film thickness and 'a' is the value of x at which $G_2(x)$ changes sign For this profile also δW is negative for $1 \le x \le b$ and the maximum load $\bar{W}_m$ is given by

$$\bar{W}_m = a^2 = \frac{g_2}{f_2} \qquad (9\ 28)$$

Equations(9 23) , (9 27) and (9 28) give

$$\bar{W}_m = a^2 = \frac{g_2}{f_2} = \frac{\int_1^a xF(M_o\ Y_o)\ dx + \int_a^b xF(M_o\ 1)\ dx}{\int_1^a \frac{F(M_o\ Y_o)}{x} + \int_a^b \frac{F(M_o\ Y_o)}{x}\ dx} \qquad (9\ 29)$$

and a is determined by

$$\frac{F(M_o\ 1)}{F(M_o\ Y_o)} = \frac{2a^2\ \ln a - (a^2-1)}{(b^2 - a^2) - 2a^2\ \ln \frac{b}{a}} = S_3\ , \qquad (9\ 30)$$

where the function S_3 is the same as S_2 where a_m is replaced by a

The function S_3 is plotted with respect to M_o and Y_o in figures (9 2) and (9 4) Figures (9 2) and (9 4) show that the function S_3 increases as Y_o increases or $\frac{a}{b}$ approaches unity while figure (9 3) shows that this function decreases as M increases for $Y_o > 1$ From this it is clear that $\frac{a}{b}$ increases as Y_o increases and decreases as M_o increases From equation (9 28) it is seen that the optimum load capacity is proportional to a^2 and therefore to $\frac{a}{b}$ for fixed b, and so it may be concluded that $\bar{W}_m$ increases as the step height ratio Y_o increases and decreases as M_o increases for $Y_o > 1$ Thus, it may be remarked that a step bearing would give maximum load when there is no applied magnetic field in the system

CASE III: In the above two cases we have seen that for the maximum load capacity of the bearing either the film thickness profile or the magnetic field profile should be a step function Now the question naturally arises as to what happens when both non-uniformity in magnetic field and step in film thickness are taken into account We investigate this question in the following For variable magnetic field $M(x)$ and variable film thickness $Y(x)$ the load capacity of the bearing can be rewritten as [see equation(9 14)]

$$W = \frac{\int_1^b xF(M,Y)\,dx}{\int_1^b \frac{F(M,Y)}{x}\,dx} = \frac{g_3}{f_3}, \qquad (9\ 31)$$

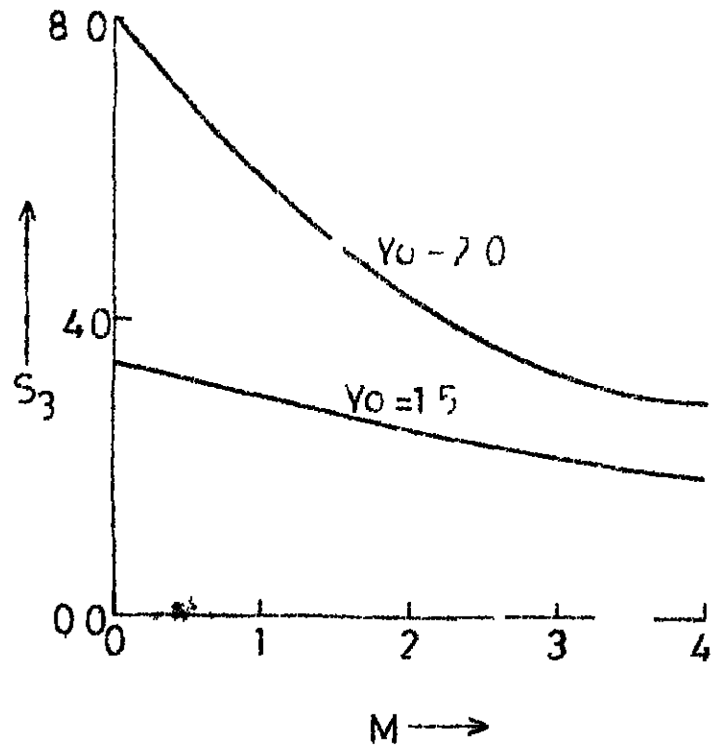


FIG 9 3 VARIATION OF S_3 WITH M FOR DIFFERENT Yo

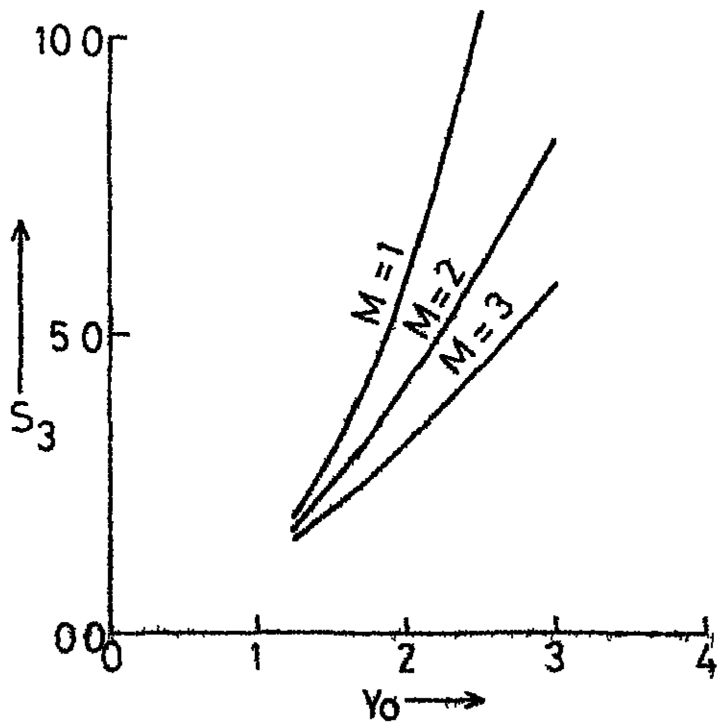


FIG 9 4 VARIATION OF S_3 WITH Yo FOR DIFFERENT M

$$g_3 = \int_1^b xF(M,Y)\,dx \qquad \text{where}$$

$$f_3 = \int_1^b \frac{F(M,Y)}{x}\,dx \qquad \text{and} \tag{9 32}$$

Now any change δM in M and δY in Y will give rise to a change $\delta\bar{W}$ in $\bar{W}$. From equation (9 31), $\delta\bar{W}$ can be written as follows:

$$\delta\bar{W} = \frac{1}{f_3^2}\int_1^b G_3(x)\left(\frac{\partial F}{\partial M}\delta M + \frac{\partial F}{\partial Y}\delta Y\right)dx \tag{9 33}$$

where
$$G_3(x) = \frac{g_3}{x} - xf_3 \tag{9 34}$$

Here also we have to consider those variations of M and Y such that the sign of $\delta\bar{W}$ is negative for maximum $\bar{W}$. Following Rayleigh [1918], the sign of $G_3(x)$ is seen to be positive when x goes to 1 and it is negative when x approaches b.

Let us consider the following profiles for magnetic field $M(x)$ and film thickness $Y(x)$

$$\begin{aligned} M &= 0\ , & 1 \le x \le a_m \\ M &= M_0 & a_m \le x \le b \\ Y &= Y_0\ , & 1 \le x \le a \\ Y &= 1\ , & a \le x \le b \end{aligned} \tag{9 35}$$

where a_m is the value of x at which $G_3(x)$ changes sign.

Let us assume that $a \le a_m$. It is seen that for the region $1 \le x \le a$, $G_3(x) \ge 0$, $\frac{\partial F}{\partial M} = 0$, $\frac{\partial F}{\partial Y} < 0$ and δY is negative which

implies that $\delta\bar{W}$ is negative Again, in the region $a_m \leq x \leq b$, since $G_3(x) \leq 0$, $\frac{\partial F}{\partial M} \geq 0$ for $M \geq 0$, $\frac{\partial F}{\partial Y} < 0$, $\delta M < 0$ and δY is positive, so that the sign of $\delta\bar{W}$ is clearly negative Let us now examine the sign of $\delta\bar{W}$ in the region $a \leq x \leq a_m$ In this range $G_3(x)$ is positive, $\frac{\partial F}{\partial M}$ is positive, δM is positive, $\frac{\partial F}{\partial Y}$ is negative and δY is positive, so the sign of $\delta\bar{W}$ may be positive or negative [see equation (9 33)] Since for maximum $\bar{W}$, $\delta\bar{W}$ is to be negative in this region also, we should take a situation which makes $\delta M > 0$ and $\delta Y < 0$ This is clearly satisfied if we make a tend to a_m The same argument holds in case $a_m \leq a$ Thus the above chosen profiles give $\delta\bar{W} < 0$ throughout the region, when $a = a_m$

Since $G_3(x) = 0$ at $x = a_m = a$, from equations (9 31) and (9 34) we have,

$$\bar{W}_m = \frac{g_3}{f_3} = a^2 \tag{9 36}$$

Now, from equations (9 32), (9 35) and (9 36) we have the expression determining a as follows

$$a^2 = \frac{1}{2}\ \frac{(a^2-1)\ F(0,Y_o) + (b^2-a^2)\ F(M_o,1)}{F(0,Y_o)\,\ln a + F(M_o,1)\ \ln\ \frac{b}{a}} \tag{9 37}$$

$$\text{or,}\quad \frac{F(M_o,1)}{F(0,Y_o)} = \frac{2a^2\ln a - (a^2-1)}{(b^2-a^2)-2a^2\ln\ \frac{b}{a}} = S_4 \tag{9 38}$$

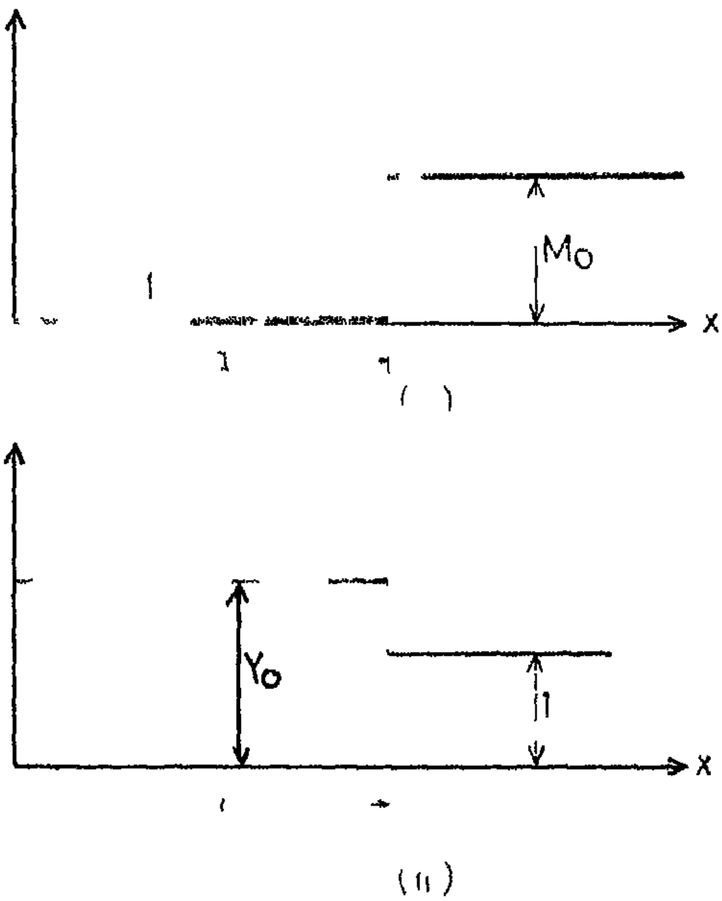
It can be again noted here, that the function $\frac{F(M_o,1)}{F(0,Y_o)}$ increases as M_o or Y_o or both increase Further, as S_4 increases as 'a' increases for given b, it is concluded, as before that $\bar{W}_m$ increases as M_o or Y_o or both increase It is remarked that this maximum load is greater than that of the previous two cases, and thus it is desirable to have a magnetohydrodynamic externally pressurised bearing with stepped film thickness and the magnetic field should be uniformly applied only in the minimum film thickness region [see figure no (9 5)]

9 3 LOAD CAPACITY WHEN THE FLOW RATE Q IS PRESCRIBED:

In this case the load capacity W is written from equations (9 11) and (9 14) as

$$W = \frac{\mu Q\, r_1^2}{4h_o^3} \int_1^b \frac{x\, Y^3(x)}{\frac{M(x)Y(x)}{2} - \tanh \frac{M(x)Y(x)}{2}}\, dx \qquad (9{,}39)$$

Since $F(M,Y)$ increases as M increases for given Y, it may be inferred from equations (9 11) and (9 39) that for a given Q, both $p_i - p_a$ and W increase with increase of M It is further observed that since $\frac{MY}{2} - \tanh \frac{MY}{2}$ increases as Y increases for fixed M, W increases as Y decreases i e the step height ratio approached unity Thus, for a given flow flux, step in the film thickness is not desirable This result is true even when there is no magnetic field For Y = 1 (constant) we obtain all the results from here as investigated by Hughes and Elco [1962]



[illegible] FOR MAGNETIC FIELD

[illegible] FOR FILM THICKNESS RATIO

9 4 CONCLUSIONS

In the light of the above analysis the following conclusions may be drawn

(1) At a constant applied pressure, the magnetic field should be nonuniform for optimum load capacity if the film thickness is constant This maximum load increases as the strength of the magnetic field M_o increases

(2) For uniformly applied magnetic field M_o, the film thickness should be a step function if the load capacity is to be maximum and this maximum load capacity increases as the step height ratio Y_o increases and it decreases as M_o increases for $Y_o > 1$ Thus, for maximum load capacity it is desirable not to apply any magnetic field in the case of step bearing

(3) It is seen in Case III that, for the maximum load capacity of the bearing, both the film thickness and the magnetic field functions should be step functions having the same step location [see fig no (9 5)] This maximum load increases as M_o or Y_o or both increase

(4) When the flow rate of the lubricant is constant, both the recess pressure and load capacity increases with increase in the strength of the magnetic field Also the load capacity decreases as the film thickness ratio increases

CHAPTER - X

MAGNETOHYDRODYNAMIC EXTERNALLY PRESSURISED POROUS THRUST BEARING

In Chapter IX, the magnetohydrodynamic externally pressurized bearing has been studied when external pressure is applied at a central recess In this chapter, we study the characteristics of magnetohydrodynamic thrust bearings where one of the surfaces is porous For the sake of simplicity the porous matrix has been assumed to consist of a system of capillaries of very small radii which restrict the lubricant to flow only in the direction normal to the plates This assumption seems to be reasonable due to recent advances in ceramic technology, Hsing [1971] A constant magnetic field has been applied throughout the bearing system in the transverse direction which modifies the pressure in the film Since the applied magnetic field is in the same direction as the flow of the lubricant in the porous matrix, the velocity of the lubricant in the matrix remains unaffected and is still to be governed by the usual Darcy's law, Hsing [1971]

Here we study the following two cases of MHD porous thrust bearings

(i) MHD porous thrust infinite plates,

(ii) MHD porous thrust circular plates

Similar nonmagnetic cases have been studied by Hsing [1971], and Mori et al [1965]

10 1 MHD POROUS THRUST PLATES:

Consider the flow of an incompressible lubricant between two insulating parall l plates in the presence of a transvers ly applied magnetic field The upper plate is porous through which the lubricant is pressurised into the bearing and thus generating a symmetric l pressure dis ribution in the film The physical situation is shown in figure (10 1)

The basic equation governing the flow of the conducting lubricant under the usual hydromagnetic assumptions, Kuzma [1965] , is

$$\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{h^2} u = \frac{1}{\mu}\frac{dp}{dx} , \tag{10 1}$$

where $M = Bh\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$ is the Hartmann number The boundary conditions for the velocity are

$$\begin{aligned} u &= 0 \quad \text{at} \quad y = 0 \\ u &= 0 \quad \text{at} \quad y = h \end{aligned} \tag{10 2}$$

Solving equation (10 1) and using (10 2) we get

$$u = \frac{h^2}{\mu M^2}\frac{dp}{dx}\left[\left\{\cosh\left(\frac{My}{h}\right) - 1\right\} - \frac{\cosh M - 1}{\sinh M}\sinh\frac{My}{h}\right] \tag{10 3}$$

The flow flux Q for width b is defined by

$$Q = b\int_0^h u\,dy , \tag{10 4}$$

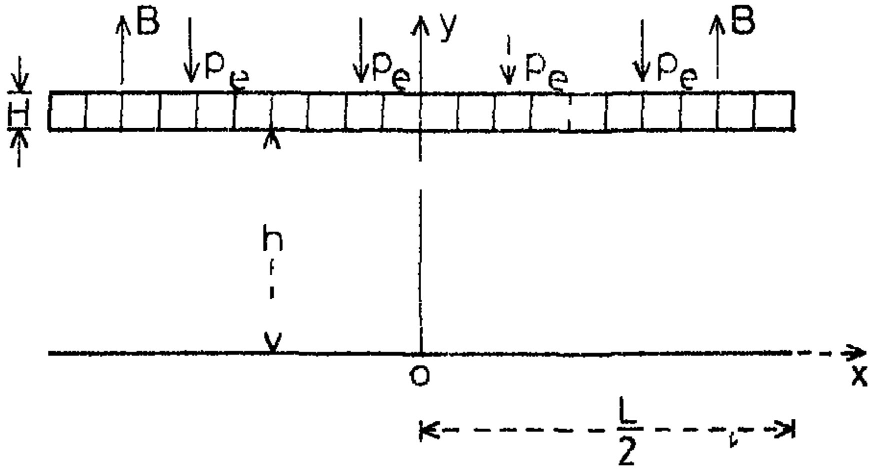


FIG 10 1 MHD - POROUS THRUST PLATES

which on using equation (10 3) gives

$$Q = \frac{2h^3}{\mu M^3} \frac{dp}{dx} \left[\tanh \frac{M}{2} - \frac{M}{2}\right] \tag{10 5}$$

The equation of continuity is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10 6}$$

Integrating equation (10 6) and using (10 4) and the conditions,

$$\begin{aligned} v &= 0 \quad \text{at} \quad y = 0 \\ v &= -v' \quad \text{at} \quad y = h \end{aligned} \tag{10 7}$$

where v' is the velocity in the porous mesh, we get

$$\frac{\partial}{\partial x} \left(\frac{Q}{b}\right) = v' \tag{10 8}$$

From equations (10 5) and (10 8) we get

$$\frac{d}{dx} \left\{\frac{2h^3}{\mu M^3} \frac{dp}{dx} \left(\tanh \frac{M}{2} - \frac{M}{2}\right)\right\} = v' \tag{10 9}$$

The velocity v' in the porous mesh is given by Darcy's law as [Hsing (1971)],

$$v' = \frac{\phi}{\mu} \frac{p_e - p}{H} \tag{10,10}$$

where ϕ is the permeability, μ is the viscosity of the lubricant and H is the thickness of the porous matrix From equations (10 9) and (10 10) we obtain

$$\frac{d}{dx}\left[\frac{2h^3}{\mu' 3}\frac{dp}{dx}\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\right] = \frac{\phi}{\mu'}(p_e - p) \qquad (10.11)$$

Solving equation (10.11) and using the conditions

$$\frac{dp}{dx} = 0 \quad \text{at} \quad x = 0$$

$$p = 0 \quad \text{at} \quad x = \frac{L}{2}, \qquad (10.12)$$

we get

$$p = p_e\left(1 - \frac{\cosh\frac{2\beta x}{L}}{\cosh\beta}\right), \qquad (10.13)$$

where $$\beta^2 = \frac{\phi L^2}{8Hh^3}\,\frac{M^3}{\frac{M}{2} - \tanh\frac{M}{2}} \qquad (10.14)$$

The load capacity W of the bearing for width b is given by

$$W = 2b\int_0^{L/2} p\,dx$$

which on using equation (10.13) gives

$$\bar{W} = \frac{W}{bLp_e} = 1 - \frac{\tanh\beta}{\beta} \qquad (10.15)$$

Since

$$\frac{d}{d\beta}\left(1 - \frac{\tanh\beta}{\beta}\right) = \frac{\sinh 2\beta - 2\beta}{2\beta^2\cosh^2\beta} \qquad (10.16)$$

is positive for $\beta > 0$, therefore $1 - \frac{\tanh \beta}{\beta}$ increases as β increases Further from equation (10 14) it can be seen, as $\frac{M^3}{\frac{M}{2} - \tanh \frac{M}{2}}$ increases as M increases, that β increases as M or ϕ increases Thus it can be concluded from equation (10 15) that $\bar{W}$ increases as M or ϕ or both increase

By putting $M = 0$ in equation (10 15) we get the following expressions for the load capacity for the non-magnetic case which is the same as obtained by Hsing [1971] when inertia effects are neglected in his case

$$\bar{W}_o = 1 - \frac{\tanh \beta_o}{\beta_o} , \qquad (10\ 17)$$

where $\beta_o^2 = \frac{3\phi L^2}{Hh^3}$

For $M = 10$ and $\beta_o = 5$, the load increase from the non-magnetic case is about 17%

10 2 SQUEEZING EFFECT

Now consider the above case when the upper plate is having a relative normal velocity V due to loading, then the conditions (10 7) are modified to

$$\begin{aligned} v &= 0 \quad \text{at} \quad y = o \\ v &= -V - v' \quad \text{at} \quad y = h \end{aligned} \qquad (10\ 18)$$

Then following the same procedure as before the expressions for pressure and load capacity can be written as,

$$p = (p_e + \frac{\mu HV}{\phi})\,(1 - \frac{\cos\frac{2\beta x}{L}}{\cos\beta}) \tag{10.19}$$

$$\bar{W}_v = \frac{W}{p h_e L} = (1 + \frac{\bar{V}}{\bar{\phi}})(1 - \frac{\tanh\beta}{\beta}) \tag{10.20}$$

where $\bar{\phi} = \frac{\phi}{H^2}$, $\bar{V} = \frac{\mu V}{p_e H}$ and $W_v = (p_e + \frac{\mu HV}{\phi})\quad(1 - \frac{\tanh\beta}{\beta})\ bL$

Now from equation (10.20) it can be seen that load capacity increases as $\bar{V}$ or M increases. The squeezing time t for reducing the film thickness from h_1 to h_2 is given by

$$\bar{t} = \frac{t p_e L}{\mu h_2} = \frac{1}{\bar{\phi}} \int_1^{\bar{h}_1} \frac{d\bar{h}}{\frac{\bar{W}_v\,\beta}{\beta - \tanh\beta} - 1}, \tag{10.21}$$

where $\bar{h}_1 = \frac{h_1}{h_2}$ and $\bar{h} = \frac{h}{h_2}$

Since $1 - \frac{\tanh\beta}{\beta}$ is an increasing function of β it can be seen from equation (10.21) that $\bar{t}$ increases as M increases for a given $\bar{\phi}$. When $\bar{\phi} = 0$, the expressions for $\bar{W}$ and $\bar{t}$ are the same as obtained by Shukla [1965] for large and small Hartmann numbers.

To see the effect of ϕ only, we consider the case when $M = 0$. In this case the expressions for $\bar{W}$ for small $\bar{\phi}$ is given by

$$\bar{W}_v \approx \frac{K}{3}\,[\bar{V} + (1 - \frac{2\bar{V}}{5}K)\,\bar{\phi}] \tag{10.22}$$

where $$K = \frac{3HL^2}{h^3}$$

From equation (10 21), the approximate value of t in the form of an integral is given by

$$t = \frac{\mu}{p_e} \int_{2}^{\eta_1} \frac{}{\frac{3' V}{K} - \left(1 - \frac{6'}{}\right)\phi} \qquad (10\ 23)$$

From equations (10 22) and (10 23) it may be concluded that $\bar{W}$ increases as $\bar{\phi}$ increases provided $V < 5/6 \, \frac{p_e a^3}{\mu L^2}$ which may be satisfied by a suitable choice of p_e

Again, from equation (10 23) it may be inferred that t increases as $\bar{\phi}$ increases provided $\bar{W}_v < \frac{5}{6}$ When M is very large, from equations (10 14) and (10 21) we have, $\bar{t} \sim \frac{1}{\bar{\phi}} \, \frac{(\bar{h}_1 - 1)}{\bar{W}_v - 1}$, which implies that $\bar{t}$ decreases as $\bar{\phi}$ increases

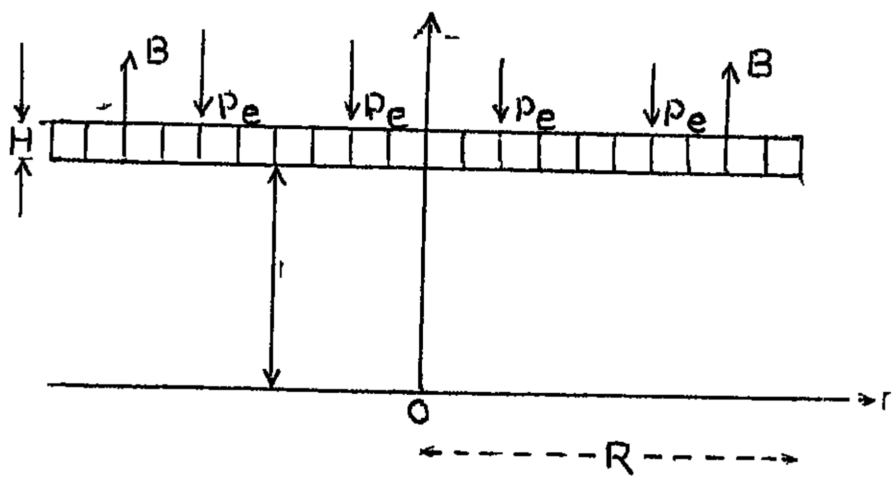
10 3 MHD POROUS THRUST CIRCULAR PLATES

In this case also the upper plate is porous and the lubricant is pressurised into the bearing which generates a symmetrical pressure distribution The physical configuration is illustrated in figure no (10 2)

The basic equation governing the flow of the incompressible lubricant under the lubrication assumptions, Hughes and Elco [1962], is given by

$$\frac{\partial^2 u}{\partial z^2} - \frac{M^2}{h^2} u = \frac{1}{\mu} \frac{dp}{dr} , \qquad (10\ 24)$$

where M is the Hartmann number, u is the velocity in the radial direction and $\frac{dp}{dr}$ is a function of r only The boundary conditions



F[illegible] MH[illegible] [illegible]HRUST [illegible]

for the velocity u ar

$$u = 0 \quad \text{at} \quad z = 0$$
$$u = 0 \quad \text{at} \quad z = h \tag{10 25}$$

Solving equation (10 24) with the help of the conditions(10 25) we obtain

$$u = \frac{h^2}{\mu M^2} \frac{dp}{dr} \left[\cosh \frac{Mz}{h} - 1 - \frac{\cosh M-1}{\sinh M} \sinh \frac{Mz}{h}\right] \tag{10 26}$$

The flow flux Q is defined by

$$Q = \int_0^h 2\pi r \, udz \tag{10 27}$$

which on using equation (10 26) gives

$$Q = \frac{4\pi r h^3}{\mu M^3} \frac{dp}{dr} \left[\tanh \frac{M}{2} - \frac{M}{2}\right] \tag{10 28}$$

The equation of continuity in this case is given by

$$\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial z} = 0 \tag{10 29}$$

Integrating equation (10 29) with respect to z and using equation (10 27) and the conditions

$$v = 0 \quad \text{at} \quad z = 0$$
$$v = -v' \quad \text{at} \quad z = h \tag{10 30}$$

we get

$$\frac{\partial Q}{\partial r} = 2\pi r v' \tag{10 31}$$

where v' is the velocity of the lubricant in the porous matrix

From equations (10 28) and (10 31) we get

$$\frac{4\pi h^3}{\mu' M^3}\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\frac{d}{dr}\left(r\frac{dp}{dr}\right) = 2\pi r\, v' , \tag{10 32}$$

The velocity v' in the porous matrix is again, given by

$$v' = \frac{\phi}{\mu}\,\frac{p_e - p}{H} \tag{10 33}$$

From equations (10 32) and (10 33) we get the final equation determining the pressure as

$$\frac{4\pi h^3}{\mu' M^3}\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\frac{d}{dr}\left(r\frac{dp}{dr}\right) = \frac{2\pi r\,\phi}{\mu}\,\frac{(p_e - p)}{H} \tag{10 34}$$

Equation (10 34) can be written as

$$\xi^2\frac{d^2P}{d\xi^2} + \xi\frac{dP}{d\xi} - \xi^2 P = 0 \tag{10 35}$$

where $P = p_e - p$, $\alpha^2 = \dfrac{\phi}{2Hh^3}\,\dfrac{M^3}{\frac{M}{2} - \tanh\frac{M}{2}}$ and $\xi = \alpha r$

The boundary conditions for p are

$$\begin{aligned} \frac{dp}{dr} &= 0 \quad \text{at} \quad r = 0 \\ p &= 0 \quad \text{at} \quad r = R \end{aligned} \tag{10 36}$$

or equivalently,

$$\begin{aligned} \frac{dP}{d\xi} &= 0 \quad \text{at} \quad \xi = 0 \\ P &= p_e \quad \text{at} \quad \xi = \alpha R \end{aligned} \tag{10 36$'$}$$

The solution of equation (10 35) can be written as

$$P = A\, I_0(\xi) + B\, K_0(\xi) \tag{10 37}$$

where A and B are constants and $I_0(\xi)$ and $K_0(\xi)$ are modified Bessel function of ord r zero of first and second kind respectively Since the pressure is finite at $\xi = 0$ and $K_0(\xi)$ has an infinite singularity at $\xi = 0$, we get $B = 0$ Then from equation (10 37) we have

$$P = A\, I_0(\xi) \tag{10 38}$$

After determining the constant A from the second boundary condition (10 36) the expression for pressure p can be written as

$$p = p_e\left[1 - \frac{I_0(\xi)}{I_0(\beta_1)}\right], \tag{10 39}$$

where $\beta_1 = \alpha R$

The load capacity W is given by

$$W = \int_0^R 2\pi r\, p\, dr \tag{10 40}$$

From equations (10 39) and (10 40) we get

$$\bar{W} = \frac{W}{\pi p_e R^2} = \left[1 - \frac{2 I_1(\beta_1)}{\beta_1 I_0(\beta_1)}\right] \tag{10 41}$$

The variation of the function $1 - \frac{2 I_1(\beta_1)}{\beta_1 I_0(\beta_1)}$ is shown in figure no (10 3), from which it can be seen that this function increases as

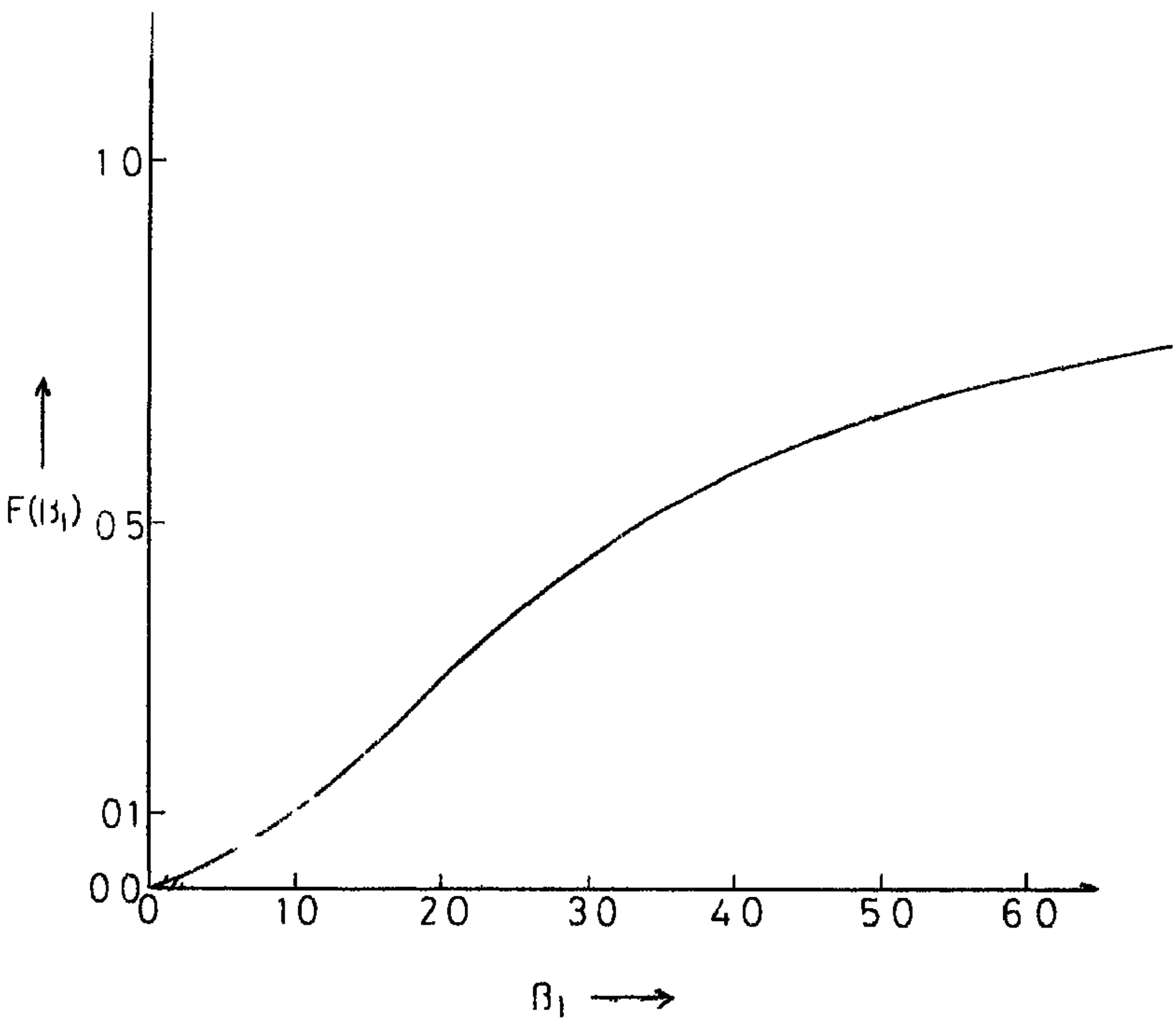


FIG 10 3 VARIATION OF $F(\beta_1) = 1 \; \frac{2 I_1(\beta_1)}{\beta_1 I_0(\beta_1)}$ WITH β_1

β_1 increases Since $\beta_1^2 = \frac{\phi R^2}{2Hh^3} \frac{M^3}{\frac{M}{2} - \tanh\frac{M}{2}}$ increase as ϕ or M or both increase, therefore it can be concluded from equation (10 41) that $\tilde{W}$ increases as ϕ or M or both increase

10 4 SQUEEZING EFFECT

We now consider the above case when there is a relative normal velocity V between the plates The conditions (10 30) are now modified to

$$\begin{aligned} v &= 0 \quad \text{at} \quad z = 0 \\ v &= -V-v' \quad \text{at} \quad z = h \end{aligned} \tag{10 42}$$

Proceeding as before the expressions for pressure and load capacity are given by

$$p = (p_e + \frac{\mu HV}{\phi}) [1 \quad \frac{I_o(\xi)}{I_o(\beta_1)}] \tag{10 43}$$

and

$$W_v = \pi(p_e + \frac{\mu HV}{\phi}) R^2 [1 \quad \frac{2 I_1(\beta_1)}{\beta_1 I_o(\beta_1)}] \tag{10 44}$$

Making dimensionless, we have from equation (10 44)

$$\tilde{W}_v = \frac{W_v}{\pi p_e R^2} = (1 + \frac{\tilde{V}}{\phi}) [1 - \frac{2I_1(\beta_1)}{\beta_1 I_o(\beta_1)}] \tag{10 45}$$

It is seen from equation (10 45) as before that the load capacity increases as $\tilde{V}$ or M increases

The squeezing time t for reducing the film thickness from h_1 to h_2 is given by

$$\bar{t} = \frac{h_e\ l}{\mu h_2}\, t = -\frac{1}{\bar{\phi}} \int_{1}^{h_1} \frac{dh}{\dfrac{\bar{W}_v}{1 - \dfrac{2I_1(\beta_1)}{\beta_1 I_0(\beta_1)}} - 1} , \tag{10.46}$$

where $I_0(\beta_1)$ and $I_1(\beta_1)$ are all functions of h Since $1 - \frac{2I_1(\beta_1)}{\beta_1 I_0(\beta_1)}$ is an increasing function of β_1 (see fig 10 3) it is seen from equation (10 46) that $\bar{t}$ increases as M increases for a given $\bar{\phi}$ By taking $\bar{\phi} = 0$ in the above expressions for $\bar{W}$ and $\bar{t}$ we get all the results as obtained by Shukla [1965] for large and small Hartmann numbers

In order to see the effect of $\bar{\phi}$, the corresponding load $\bar{W}_v$ for small β_1 and M can be written as follows

$$\bar{W}_v \approx \frac{K_1}{8}\left[\bar{V} + \left(1 - \frac{K_1 \bar{V}}{6}\right)\ \bar{\phi}\right] \tag{10 47}$$

where $\beta_1^2 = K_1 \bar{\phi}$ and $K_1 = \frac{HR^2}{2h^3}\ \frac{M^3}{\frac{M}{2} - \tanh\frac{M}{2}}$ It may be seen from equation (10 47) that $\bar{W}_v$ increases as $\bar{\phi}$ increases provided $V < \frac{12h^3 p_e}{\mu R^2}\ \frac{\frac{M}{2} - \tanh\frac{M}{2}}{M^3}$ and this condition may be satisfied by suitably adjusting the applied pressure p_e

Again for small β_1, $\bar{t}$ is given from equation (10 46) as

$$\bar{t} = \int_1^{\bar{h}_1} \frac{d\bar{h}}{\frac{8\bar{W}_V}{K_1} \quad (1 - \frac{4\bar{W}_V}{3})\phi} \tag{10 48}$$

Since $\bar{h}_1 > 1$, the integrand and hence the integral increases as $\bar{\phi}$ increases provided $\bar{W}_V < \frac{3}{4}$ This implies that the time of approach is delayed due to increase in $\bar{\phi}$ provided $\bar{W}_V < \frac{3}{4}$

Further for large M and hence β_1, from equation (10 46), $\bar{t}$ can be approximated to

$$\bar{t} = \frac{1}{\phi} \frac{(\bar{h}_1 - 1)}{\bar{W}_V - 1}$$

and decreases as $\bar{\phi}$ increases for a given $\bar{W}_V$

10 5 CONCLUSIONS:

In this chapter, we have studied the effects of transversely applied magnetic field on porous thrust bearings by considering both rectangular and circular configurations It has been shown in these cases that the load capacity increases as supply pressure, permeability and the strength of the applied magnetic field increase The effects of squeezing have also been studied in the above mentioned cases and it has been pointed out that the load capacity increases as squeeze velocity increases and this increase is enhanced as the strength of the applied magnetic field increases for a fixed ϕ It has further been pointed out that the time of approach also increases as the

strength of the applied magnetic field increases for a prescribed ϕ. In the case when squeezing is taking place, the load capacity increases as ϕ increases provided certain restrictions are placed on $\bar{V}$ as has been pointed out earlier. Similarly the time of approach also increases as ϕ increases for a given M subject to certain conditions on $\bar{W}_v$.

REFERENCES

Agarwal, V K (1970)	Hydromagnetic pivoted slider bearing with a convox pad surface Jap J App Phy Vol 9, No 8, p 997
Agarwal, V K, *Ganju*, K L and Jethi, S C (1972)	*Squeeze film and externally pressurized* bearings micropolar fluid lubricated Wear 19, p 259
Allen, S J and Kline, K A (1971)	Lubrication theory for micropolar fluids J App Mech, Trans, ASME, Vol 38, p 646
Balram, M and Shastry, V U K (1972)	Micropolar lubrication J App Mech Trans, ASME, p 834
Bartz, W J (1971)	Solid lubricant additives - Effects of concentration and other additives on antiwear performances Wear 17, p 421
Batra, R L (1973)	Lubrication of a rotating circular step bearing with Bingham plastic lubricants *Defence Science Journal, Vol 23, No 3, p* 101
Bird, R B, Stewart, W E and Lightfoot, E N (1960)	Transport phenomena John Wiley and Sons, Ince
Braithwaite, E R (1967)	Lubrication and Lubricants Elsevier Publishing Company, New York
Brazier, A.D, Pitchford, P G (1965)	New additives for lubricants : From conception to consumer Proc of the Ind Lub Symp London
Cameron, A (1966)	Principles of lubrication Longmann, London
Carr, D W, Knight, R E, Gelder, R (1965)	The widening field for aerosol lubrication in the U K *Proc of the Ind Lub Symp London*
Castelli, V, Rightmire, G K and Fuller, D D (1966)	On the analytical and experimental investigation of a hydrostatic, axisymmetric complaint-surface thrust bearing Trans ASME, Paper no 66-Lub -17
Chawla, S S (1966)	The magnetohydrodynamic inclined slider bearing Jap J App Phy Vol 5, No 3, p 234
Chong, H S, Chow, C Y and Wilcock, D F (1968)	Behaviour of hydrostatic and hydrodynamic non-contacting face seals J Lub Tech Trans ASME, p 510
Chow, C Y. (1969)	Effects of angular inertia of lubricant in MHD hydrostatic thrust bearing App Sci. Res., Vol 20, No 1, p 40

Chow, T S and Saibel, E (1971) — The elastohydrodynamic problem with a viscoelastic fluid
J Lub Tech Trans ASME , p 25

Christensen, H (1969) — Elastohydrodynamic theory of spherical bodies in normal appro ch
Tr ns , ASME , Paper no 69-Lub -3

Christensen, H (1971) — Some aspects of the functional influence of surface roughness in lubrication
Wear 17, p 149

Cope, W F (1949) — The hydrodynamical theory of film lubrication
Proc Roy Soc London, Vol 197, p 201

Datta, A B (1972) — Pivoted slider bearing with convex pad surface in micropolar fluid
Jap J App Physics, Vol 11, No 1, p 98

Donaldson, R R (1971) — Variable viscosity effects in hydrostatic film : The adiabatic transformation
J Lub Tech Trans ASME , Vol 93,No 1,p 151

Dowson, D (1961) — Inertia effects in hydrostatic thrust bearing
J Basic Engg Trans , ASME , p 227

Dowson, D (1962) — A generalized Reynolds equation for fluid film lubrication
Int J Mech Sci Vol 4, p 159

Dowson, D and Taylor, C M (1966) — Elastohydrodynamic lubrication of circular thrust bearings
Trans , ASME , Paper no 66-Lubs -15

Dubois, G B , Ocvirk, F W and Wehe, R L (1960) — Study of effects of non-Newtonian oil on eccentricity ratio of a plain journal bearing
NASA Technical Note, D427

Dudzinsky, S J , Young, F J and Hughes, W.F (1968) — On the load capacity of the magnetohydrodynamic journal bearing
J Lub Tech Trans , ASME,, p 139

Dyer, D F (1969) — A numerical solution for the single screw extrusion of a polymer melt
AIChE Journal, Vol 15, p 823

Eloo, R A and Hughes, W F (1962) — Magnetohydrodynamic pressurisation of liquid metal bearings
Wear, Vol 5, p 198

Eringen, A C (1965) Theory of Micropolar Continua Development in mechanics, Vol 3, Part I, p 23

Eringen, A C (1966) Theory of Micropolar fluids J Maths and Mech Vol 16, No 1, p 1

Fucks, W and Uhlenbush, J (1962) Magnetohydrodynamic theory of lubrication Phys of Fluids, vol 5, p 498

Gansheimer, J (1972) On the lubricating properties of mixtures of mineral oil with certain inorganic phosphates, hydroxides and sulfide ASLE Trans , 15, p 201

Ghosh, T P and Mishra, S P (1968) Idealized slider bearing with Maxwell liquid as lubricants Inst of Engrs (India) J , Vol 48, No 11, p 705

Gould, P (1967) Parallel surface squeeze film The effect of variation of viscosity with temperature and pressure J Lub Tech Trans ASME, Vol 89, p 375

Gould, P (1971) High pressure spherical squeeze film J Lub Tech Trans ASME , p 207

Herzig, H L (1959) Effect of solid additives on fluid lubrication Lubrication Engineering 15, p 152

Horowitz, H H and Steidler, F E (1960) The calculated journal bearing performance of polymer thickened lubricants ASLE Trans , Vol 3, p 124

Horowitz, H H and Steidler, F E (1961) Calculated performance of non-Newtonian lubricants in finite width journal bearing ASLE Trans , Vol 4, p 275

Hsing, F C (1971) The effect of fluid inertia on a porous thrust plate-An analytical solution J Lub Tech Trans ASME , p 202

Hsu, Y C and Saibel, E (1965) Slider bearing performance with a non-Newtonian lubricant ASLE Trans , Vol 8, p 191

Hsu, Y C (1966) — Non-Newtonian flow in infinite length full journal bearing
Trans ASME, Paper no 66-LUB-D

Hudimoto, B and Tokuoka, T (1969) — Two dimensional shear flow of micropolar fluids
Int J Engg Sci, p 515

Hughes, W F and Elco, R A (1962) — Magnetohydrodynamic lubrication flow between parallel rotating disks
J Fluid Mech, Vol 13, p 21

Hughes, W F and Elco, R A (1962a) — The magnetohydrodynamic journal bearing
J Am Rocket Soc, Vol 32, p 776

Hughes, W F (1963) — The magnetohydrodynamic inclined slider bearing with transverse magnetic field
Wear, Vol 6, p 315

Hung, D and Muster, D (1970) — Non-Newtonian flow in infinite length step-shaped bearing
Trans ASME, Paper no 70-LUB-24

Kamiyama, S (1969) — Inertia effects in MHD hydrostatic thrust bearing
Trans ASME, Ser F, Vol 91, No 4, p 589

Kamiyama, S (1969a) — MHD journal bearing (Report I)
J Lub Tech Trans ASME, p 380

Kamiyama, S (1970) — Inertia effects in MHD hydrostatic thrust bearing
The Reports of the Institute of High Speed Mechanics, Tohoku University, Sendai, Japan

Kamiyama, S (1971) — Influence of wall conductance on the performance of the MHD hydrostatic thrust bearing
Trans ASME, Ser F, Vol 93, p 113

Kamiyama, S and Sato, A (1971) — MHD journal bearing (Report 2)
The Reports of the Institute of High Speed Mechanics, Tohoku University, Japan

Kamiyama, S and Sato, A (1972a) — The effect of wall conductance on Torque of the MHD viscous coupler and hydrostatic thrust bearing
Trans ASME, Paper no 72-LUB 1

Kamiyama, S and Sato, A (1972b) Magnetohydrodynamic flow between parallel rotating disks (Report-I, Influence of wall conductance)
Bulletin of JSME, Vol 15, No 86, p 941

Kapur, V K (1968) Centrifugal effects in magnetohydrodynamic pressurization of liquid metal bearings
Jap J pp Phy , Vol 7, No 9, p 969

Kapur, V K (1969) Magnetohydrodynamic pivoted slider bearing with a convex pad surface
Jap J App Phy , Vol 8, No 7, p 827

Kauzlarich, J J and Greenwood, J A (1972) Elastohydrodynamic lubrication with Herschel-Bulkley model greases
ASLE Trans , Vol 15, No 4, p 269

Khader, M S and Vachon, R I (1973) Theoretical effects of solid particles in hydrostatic bearing lubricant
J Lub Tech Trans ASME , p 104

Krieger, R J , Day, H J and Hughes, W F (1967) The MHD hydrostatic thrust bearing theory and experiments
J Lub Tech Trans ASME, Vol 89, p 307

Kuzma, D C (1963) MHD journal bearing
J Basic Engg Trans ASME , Vol 85, p 424

Kuzma, D C (1964a) MHD squeeze films
J Basic Engg Trans ASME , Vol 86, p 441

Kuzma, D C (1964b) The finite magnetohydrodynamic journal bearing
J Basic Engg Trans ASME , Vol 86, p 445

Kuzma, D C , Maki, E R and Donnelly, R J (1964) The magnetohydrodynamic squeeze film
J Fluid Mech , Vol 19, No 3, p 395

Kuzma, D C (1965) The magnetohydrodynamic paralle slider bearing
J Basic Engg Trans ASME , Vol 87, p 778

Kuzma, D C (1967) Fluid inertia effects in squeeze film
App Sci Res 18, p 15

Leitmann, G (1962)	Optimization techniques with application to aerospace systems Academic Press, New York
Longwell, P A (1966)	Mechanics of fluid flow McGraw Hill Book Co New York
Maday, C J (1968)	A bounded variable approach to optimum slider bearing J Lub Tech Trans ASME, Vol 90, p 144
Maki, R E, Kuzma, D C and Donnelly, R J (1966)	Magnetohydrodynamic lubrication flow between parallel plates J Fluid Mech, Vol 26, No 3, p 357
Maki, R E, Kuzma, D C and Donnelly, R J (1967)	The magnetohydrodynamic thrust bearing J Fluid Mech Vol 30, Part I, p 83
McCabe, J T (1965)	Molybdenum disulfide-its role in lubrication A brief summary of the state of the Art Proc Ind Lub Symp, London
Milne, A A (1953)	Theory of grease lubrication of slider bearing Proc Second Int Congress of Rheology, P 427
Milne, A A (1957)	On grease lubrication of a slider bearing Conf on lubrication and Wear (Institution of Mechanical Engineers, London), Paper no 102
Milne, A A (1957a)	Theory of Rheodynamic lubrication for a Maxwell liquid Conference on Lubrication and Wear, London, paper no 41
Milne, A A (1963)	Inertia effects in self acting bearing lubrication theory Proc Int Symp on Lubrication and wear, University of Houston, Texas, p 423
Molyneux, P H (1967)	Chemical additives Lubrication and Lubricants by Braithwaite, Elsevier Publishing Co, N Y
Mori, H, Yabe, H and Ono, T (1965)	Theory of externally pressurised circular thrust porous gas bearing J Basic Engg Trans ASME, Vol 87, No 3, p 613

Ng C W and Saibel, E (1972) Non-linear viscosity effects in slider bearing lubrication
J Basic Engg Trans ASME , vol 84,p 192

Osterle, F and Saibel, E (1955) On the effect of lubricant inertia in hydrodynamic lubrication
ZAMP, Vol 6, p 334

Osterle, F ,Charnes, A and Saibel, E (1955) Rheodynamic squeeze film
Presented at the 1955 annual meeting of the American Society of Mechanical Engineering

Osterle, F and Young, F J (1962) On the load capacity of hydromagnetically lubricated slider bearing
Wear, Vol 5, p 227

Pinkus, O and Sternlicht, B (1961) Theory of hydrodynamic lubrication
McGraw Hill Book Co Inc New York

Prakash, J (1967) Magnetohydrodynamic effects in composite bearing.
Trans ASME , ser F , Vol 89, p 323

Ramanaiah, G (1966) Magnetohydrodynamic step slider bearing with variable viscosity
App Sci Res Sec B, Vol 12, p 424

Ramanaiah, G (1967) Inertia effects in hydromagnetic lubrication-1 Externally pressurized bearing
App Sci Res Section A, Vol 17, p 296

Rayleigh, Lord (1918) Notes on theory of lubrication
Phil Mag Vol 35, p 1

Reynolds, O (1886) On the theory of lubrication and its application to Beauchamp Tower's experiments
Phil Trans Roy Soc.,London, 177, pt I, p 157

Rodkiewicz, C M and Anwar, M I (1971) Inertia and convective effects in hydrodynamic lubrication of a slider bearing
J Lub Tech Trans , ASME, p 313

Rodkiewicz, C M and Anwar, M I (1972) Non-uniform magnetic field effects in MHD slider bearing
J Lub Tech Trans ASME , p 101

Rodkiewicz, C M and Anwar, M I (1972) Solution for MHD slider bearing for arbitrary magnetic field
J Lub Tech Trans ASME , p 288

Rottem, Z and Shinnar, R (1962) Non-Newtonian flow between parallel boundaries in linear movement-II Non-isothermal flow
Chemical Engg Sci , Vol 17, p 53

Rowe, C N (1970) Role of additive adsorption in the mitigation of wear
ASLE Trans , Vol 13, p 179

Rutgers, I R (1962) Relative viscosity of suspensions of rigid spheres in Newtonian liquids
Rheologica Acta, Vol 2, No 3, p 202

Sasaki, T , Mori, H and Okino, N (1962) Fluid lubrication theory of roller bearing
J Basic Engg Trans ASME , Vol 84, p 166

Seshadri, V and Sutera, S P (1968) Concentration changes of suspensions of rigid spheres flowing through tube
J of Colloid and Interface Science, Vol 27, No 1, p 101

Selby , T W (1958) The non-Newtonian characteristics of lubricating oils
ASLE Trans , Vol 1, p 71

Sherman, P (1962) The viscosity of emulsions
Rheologica Acta, Vol 2, No. 1, p 74

Shukla, J B (1963) Principles of hydromagnetic lubrication
J of Phy Soc of Japan, Vol 8, p 1086

Shukla, J B (1963a) Conical step bearing using a power law lubricant
Wear, Vol 6, p 371

Shukla, J B (1964a) The magnetohydrodynamic composite slider bearing
Wear, Vol 7, p 460

Shukla, J B (1964b) Theory for the squeeze film for power law lubricants
Trans ASME , Paper no 64-Lub-4

Shukla, J B (1964c) Load capacity and time relation for squeeze films in conical bearings
Wear, vol 7, p 368

Shukla, J B (1965) — Hydromagnetic theory of squeeze films
J Basic Engg Trans ASME , Vol 87, p 142

Shukla, J B (1965a) — Magnetohydrodynamically lubricated externally pressurised bearing with variable film thickness
App Sci Res , Sec B, Vol 11, No 6, P 453

Shukla, J B and Prasad, R (1965) — Hydromagnetic squeeze film between two conducting surfaces
J Basic Engg Trans ASME , Vol 87, p 818

Shukla, J B and Prakash, J (1966) — Hydromagnetic squeeze film between two finite plates
Jap J App Phy , Vol 5, No 4, p 322

Shukla, J B and Prasad, R (1966) — Squeezing effects in a magnetohydrostatic circular step bearing
Bulletin of the National Institute of Science of India, No 33

Shukla, J B and Kapur, V K (1967) — Inertia effects in magnetohydrodynamically lubricated hydrostatic thrust bearing
Jap J App Phy , Vol 6, No 12, p 1449

Shukla, J B and Prasad, R (1967) — Magnetorheostatic bearing and magnetorheodynamic squeeze film
Trans ASME , 67-LUBS-4

Shukla, J B and Prakash, J (1969) — The Rheostatic thrust bearing using power law fluids as lubricants
Jap J App Phy , vol 8, No 12, p 1557

Shukla, J B (1970) — The optimum one dimensional MHD slider bearing
J Lub Tech Trans ASME , p 530

Shukla, J B (1972) — Proc First World Conference in Industrial Tribology, New Delhi, India

Shvarts, I A (1966) — Magnetohydrodynamic lubrication theory for cylindrical bearing
MeKhanika Zhidkosti i Gaza, vol 1, No 1, p 9

Sneck, H J (1969) — Thermal effects in face seal
J Lub Tech , Trans ASME, p 434

Snyder, W T (1962) The magnetohydrodynamic slider bearing J Basic Engg , Trans ASME, Vol 84, p 197

Snyder, W T (1963) The non-linear hydrodynamic slider bearing J Basic Engg , Trans ASME , Vol 85, p 429

Stock, A J (1966) Evaluation of solid lubricant dispersions on a four ball tester Lubrication Engg , 22, p 146

Talivaldis Spalvins (1971) Lubrication with sputtered molybdenum disulfide film ASLE Trans Vol 14, No 4, p 267

Tanner, R I (1960) Non-Newtonian flow and the oil seal problem J Mech Engg Sci , vol 2, No 1, p 25

Tanner, R I (1963) Non-Newtonian lubrication theory and its application to the short journal bearing Australian Journal of Applied Science, Vol 14, No 2, p 129

Tanner, R I (1964) A short bearing solution for pressure distribution in a non-Newtonian lubricant J App Mech Trans ASME., p 350

Tanner, R I (1965) Study of anisothermal short journal bearings with non-Newtonian lubricants J App Mech Trans ASME , Paper no 65-APM-M

Tanner, R I (1970) Non-Newtonian and Viscoelastic behaviour of fluids Trans ASME., Paper no 70-DE-21, p 8

Ting, L L and Mayer, JR J E (1971) The effect of temperature and inertia on the hydrostatic thrust bearing J Lub Tech Trans ASME , Vol 93, p 307

Turian, R M (1965) Viscous heating in the cone-and-plate viscometer-III Non-Newtonian fluids with temperature dependent viscosity and thermal conductivity Chem Engg Sci Vol 20, p 771

Walker, L. and Osterle, F (1961) A contrifugal effect self acting thrust bearing
ASLE Trans , Vol 4, p 287

Wright, W A and Crouse, W W Jr (1965) A new concept in generalizing non-Newtonian fluid flow data
ASLE Trans , Vol 8, p 184